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Benford's Law and Public Works Audit: an analysis of overpricing in the maracanã remodeling

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ABSTRACT

The pricing analysis in public works audits is a time consuming task and sometimes the auditor spends weeks of his work time doing it, since, in many cases, the budget spreadsheets are long and difficult to analyze. The Newcomb-Benford Law constitutes a data mining tool to be used in conjunction with the ABC curve and is supposed to contribute with a more efficient selection of the services in the spreadsheets for the overpricing analysis. Such law proposes that the frequencies of the leading digits in a multitude of databases are decreasing from 1 to 9; the digit 1 appears in, approximately, 30% of the data, while the digit 9 does not reach 5% of such values. This study aims at demonstrating the application of the Newcomb-Benford Law to the pricing spreadsheets of public works. The methodology consisted of: an introduction to the Newcomb-Benford Law and its main applications; the presentation of the relevant tests of the Law in question; the development and application of such tests to the Maracanã remodeling works; the comparison of the results obtained with the analysis carried out by the Brazilian Federal Court of Accounts (TCU); and the presentation of the results, conclusions and next research. It has been found, in a general way, that the work in question presented a marginal compliance with the Benford Law. However, the tests allowed for the identification of 17 services in the spreadsheet which did not respect the Law and repre-

sented 71.54% of the total overpricing found by TCU (R$ 149,972,318.01)

Keywords: Audit. Public Works. Newcomb-Benford Law. Overpricing.

1. INTRODUCTION

If one throws a dice at random, the odds on rolling number 5, for example, are 1/6, since the dice has six sides. When one tosses a non biased coin, there is 50% chance of getting heads or tails. Therefore, we tend to think that, in a numerical database, the chance of randomly choosing a figure whose leading digit is 1 is 1/9, and the same goes for any other figure from 2 to 9.

Simon Newcomb (1881), an astronomer and mathematician of the 19th century, observed that the pages of books of logarithms were dirtier in the beginning and progressively cleaner throughout, which indicated that, usually, 1 was the most accessed digit, and the frequency decreased up to 9. Such finding was contrary to the common understanding of uniform distribution of frequency of those digits. As Newcomb did not compile numerical data or provide any other evidence of his finding, it was only half a century later, when the physicist Frank Benford (1938) incidentally came to the same conclusion, that such fact began to gain ground. In 1938, Benford published his seminal work, The Law of Anomalous Numbers, in which he used data collected from different types of sources. Such data were random and unrelated to each other. They varied from numbers obtained from the pages of the main newspapers and all the figures of an important topic from Reader's Digest to mathematical tables and scientific constants. His work analyzed the first digits of the collected data and showed that 30.6% of the numbers had 1 as the leading digit; the first digit 2 occurred in 18.5% of the cases; and that, by contrast, only 4.7% of the numbers had 9 as the leading digit. Such first digit frequencies apply to a number of data sources, including energy bills, addresses, stock prices, population figures, mortality rate, among others. This distribution is now known as Benford's distribution. On the other hand, the property found out by Newcomb and Benford is known as the Newcomb-Benford Law or, more simply, Benford's Law.

In order to better understand the reason for the frequency differences, suppose you invest R$10,000 in a pension fund that offers a yearly fixed rate return of 7%. Your investment, then, will double in approximately every ten years. Therefore, after ten years having 1 as the leading digit, the amount of your investment will finally reach 20,000. After another ten years, the resource will double to 40,000 (in most part of these ten years the figures will begin with 2 and in the other smaller part they will begin with 3). After another decade, the amount will reach 80,000 (the numbers 4, 5, 6 and 7 will appear as first digits in only ten years). At a certain time, the amount will reach 100,000 and the leading digit 1 will appear for another ten years. In this way, when choosing a random date, it is more likely that the amount of your investment on that day will have 1 as the leading digit, rather than any other number. This same logic applies to several data in nature, such as the size of populations, among others.

A database is more likely to represent a Benford distribution if the data are collected from different distributions (Hill, 1995). On the other hand, figures assigned by human intervention, such as Social Security numbers, postal codes, bank accounts, telephone numbers or numbers fabricated by students in experiments, usually do not comply with Benford's Law (Nigrini, 2000). Such observation suggests that the Law of Anomalous Numbers can be used to detect signs of human manipulation of data.

Several studies have assumed the hypothesis that fabricated data are identified by the deviation of digits in relation to Benford's distribution. Nigrini (1992, 2000, 2012), assuming that accurate accounting data followed Benford's distribution closely (as his research showed they did), argued that substantial deviations in relation to such Law suggested possible frauds or fabricated data The author developed several tests to measure compliance with Benford's Law, and the Wall Street Journal (Berton, 1995) reported that the Brooklyn Attorney's Office, in New York, had detected frauds in seven New York companies by using such tests. In this case, the evidence provided was the finding that fraudulent and random data had a few figures beginning with 1 and many beginning with 6. Based on such previous successes, Nigrini was invited to give advice to tax agencies in several countries and to install the Benford's Law tests in most computer programs for fraud detection.

Rauch Göttsche, Brähler and Engel (2011) published a paper in the German Economic Review, in which they demonstrated that Benford's Law could be used to test macroeconomic data, revealing which ones needed to be more tightly inspected. They analyzed the compliance of the first digit of macroeconomic data reported to the Eurostat - European Union Statistical Office – by EU member countries with Benford's Law. A ranking of the 27 member countries was built, based on the extent of deviation found. The country with the greatest deviation was Greece, whose data manipulation had been confirmed by the European Commission (2010).

Walter Mebane, an American statistician from the University of Michigan, analyzed the data from the Iranian elections in 2009 and found deviations which strongly indicated the occurrence of fraud in Ahmadinejad's victory (Mebane, 2009Mebane had previously studied electoral data of several countries, including the United States, Russia and Mexico. In 2006, he found out that vote counting tended to follow Benford's Law for the second digit (Mebane, 2006). When testing the Iranian data referring to 2009, Mebane observed that, in the cities where there were not many invalid votes, Ahmadinejad's figures were a long way from Benford's distribution and that the candidate, in such situations, was well ahead in number of votes.

The research in question aims to present a case study of the application of the NB Law to public works, by using the budget spreadsheet of the Maracanã remodeling work. Such work was selected because it presented a relevant amount of data and due to the possibility of comparing the test results with the price analysis carried out by TCU. Firstly, the pertinent Benford's Law tests will be presented. Subsequently, such tests will be applied to the above mentioned spreadsheet; the comparison between the results and the TCU price analysis will be carried out and shown in the conclusion.

2. BENFORD'S LAW TESTS BASED ON THE PROBABILITIES OF THE DIGITS

The tests described below, characterized according to Negrini (2012), have been applied in this study.

2.1 THE FIRST TWO DIGITS TEST

The expected frequency of occurrence of a number D2=d2 as second digit in a set of values, given that the first digit is D1=d1, according to the NB Law, is given by:

$$\text{Prob}(D_1D_2 = \text{I}_1d_2) = \log\left(1 + \frac{1}{d_1d_2}\right) \text{(I)}$$

D_1D_2 = First two digits and $d_1d_2 \in \{10, 11,..., 99\}$

2.2 SUMMATION TEST

The Summation Test constitutes an Advanced Test developed by Nigrini (2012). When simulating a Benford distribution, he found out that the sum of the numbers in each first digit group 10, 11, 12, ..., 99 resulted in approximately similar values, that is, 1/90.

However, the author found that actual data hardly comply with such pattern, since they present very high values or many repetitions of average value figures. The usefulness of the summation test is exactly to warn about such situations.

When the Summation Test is compared to the First Two Digits Test, the data volume in each group is compared to the sum of their values. Therefore, in the case of public works budget, the materiality and the relevance of each group are observed, in order to select the digits that deserve a more rigorous critical eye by the auditor.

3. ANALYSIS OF THE MARACANÃ REMODELING WORK

The analysis of this research focused on the Maracanã remodeling budget originally presented to TCU, totaling R$ 931,885,382.19, as other budgets that virtually eliminated overpricing from most items in the spreadsheet were presented afterwards. The original budget was selected, so that the data analysis occurred in the most effective way possible, encompassing all the overpricing initially pointed out by the TCU technical unity. The research in question included only the unitary costs, however, the service costs and the total prices could also have been tested. 828 items were examined. Values under R$10.00 were excluded because they did not have a second digit.

3.1 FIRST TWO DIGITS TEST

In order to obtain a more detailed analysis, and to reduce the size of the critical digits sample, the first two digits test was carried out. The results are reported on table 1, where "Dig." refers to the first two digits of the values; "C" corresponds to the absolute frequencies with which the digits are repeated in the spreadsheet; "Real" refers to the relative frequencies with which the digits are repeated in the spreadsheet; "LB" refers to the standard frequencies of the NB Law; "Dif." means the difference between "Real" and "LB"; "Z Test" refers to the Z Test results used to measure compliance with the NB Law; "QQ." shows the results of the Chi-square test; and "MDA" presents the result of the Mean Absolute Deviations.

As table 1 shows, there were more intense peaks in the digits 11, 16, 25, 28 and 42 in relation to the proportions of the descending curve of the NB Law. The results of the Z Test were: 11 (2.954), 16 (2.105), 25 (2.524), 28 (2.303) and 42 (2.060).

Therefore, as only five of the 90 digits surpassed the limit of 1.96, it can be said that the proportions of the first digits of the Maracanã unitary costs, in general, did not deviate from the NB Law for the test in question, based on Nigrini (2012), who considers the occurrence of up to five peaks acceptable for this test.

The critical value for 89 degrees of freedom and 0.05 degrees significance is 112.02. Hence, as the calculated value in the test did not exceed the critical value, the null hypothesis cannot be rejected, which suggests compliance with the NB Law.

The Mean Absolute Deviation (MAD) test was the last one to be applied The value found for Maracanã was 0.0031, which was above 0.0022 (borderline value between compliance and non-compliance adopted by Nigrini, 2012). Such fact placed it in the non-compliance range of the reference values.

Regarding all the applied tests, it can be said that the Maracanã budget spreadsheet has passed the Mean Absolute Deviation Test for the first two digits of the unitary costs, but presented a satisfactory result in the Chi-square Test and in the Z Test.

3.2 SUMMATION TEST

The Summation Test was carried out, in a complementary way, in order to properly select the critical digits. The test in question checked the materiality of each pair of digits in the budget spreadsheet. The previous tests checked the number of repetitions in relation to the standard of the NB Law; however, they did not observe the magnitude of the services corresponding to those digits. The results are shown on table 2.

Table 1
First Two Digits Test for the Maracanã unitary costs

Díg.	C	Real	LB	Dif.	Z Test	QQ	MDA	Díg.	C	Real	LB	Dif.	Z Test	QQ	MDA
10	43	0,052	0,041	0,011	1,435	2,222	0,011	55	7	0,008	0,008	0,001	0,008	0,042	0,001
11	48	0,058	0,038	0,020	2,954	8,925	0,020	56	11	0,013	0,008	0,006	1,645	3,376	0,006
12	28	0,034	0,035	-0,001	0,054	0,021	0,001	57	10	0,012	0,008	0,005	1,303	2,244	0,005
13	26	0,031	0,032	-0,001	0,029	0,016	0,001	58	8	0,010	0,007	0,002	0,548	0,559	0,002
14	28	0,034	0,030	0,004	0,548	0,410	0,004	59	2	0,002	0,007	-0,005	1,447	2,706	0,005
15	31	0,037	0,028	0,009	1,535	2,616	0,009	60	5	0,006	0,007	-0,001	0,183	0,150	0,001
16	32	0,039	0,026	0,012	2,105	4,772	0,012	61	6	0,007	0,007	0,000	0,063	0,004	0,000
17	26	0,031	0,025	0,007	1,105	1,443	0,007	62	1	0,001	0,007	-0,006	1,780	3,927	0,006
18	18	0,022	0,023	-0,002	0,216	0,107	0,002	63	4	0,005	0,007	-0,002	0,490	0,488	0,002
19	16	0,019	0,022	-0,003	0,458	0,324	0,003	64	6	0,007	0,007	0,001	0,181	0,032	0,001
20	18	0,022	0,021	0,001	0,110	0,012	0,001	65	2	0,002	0,007	-0,004	1,280	2,219	0,004
21	14	0,017	0,020	-0,003	0,550	0,445	0,003	66	5	0,006	0,007	0,000	0,176	0,031	0,000
22	12	0,014	0,019	-0,005	0,880	0,993	0,005	67	3	0,004	0,006	-0,003	0,794	1,017	0,003
23	10	0,012	0,018	-0,006	1,240	1,838	0,006	68	1	0,001	0,006	-0,005	1,642	3,440	0,005
24	12	0,014	0,018	-0,003	0,574	0,489	0,003	69	7	0,008	0,006	0,002	0,585	0,644	0,002
25	24	0,029	0,017	0,012	2,524	6,944	0,012	70	6	0,007	0,006	0,001	0,177	0,159	0,001
26	7	0,008	0,016	-0,008	1,662	3,182	0,008	71	4	0,005	0,006	-0,001	0,237	0,211	0,001
27	10	0,012	0,016	-0,004	0,718	0,724	0,004	72	5	0,006	0,006	0,000	0,018	0,000	0,000
28	4	0,005	0,015	-0,010	2,303	5,887	0,010	73	7	0,008	0,006	0,003	0,729	0,908	0,003
29	7	0,008	0,015	-0,006	1,353	2,210	0,006	74	4	0,005	0,006	-0,001	0,149	0,142	0,001
30	13	0,016	0,014	0,001	0,208	0,124	0,001	75	4	0,005	0,006	-0,001	0,121	0,122	0,001
31	9	0,011	0,014	-0,003	0,571	0,512	0,003	76	6	0,007	0,006	0,002	0,370	0,359	0,002
32	13	0,016	0,013	0,002	0,434	0,338	0,002	77	2	0,002	0,006	-0,003	0,996	1,502	0,003
33	15	0,018	0,013	0,005	1,157	1,694	0,005	78	3	0,004	0,006	-0,002	0,506	0,546	0,002
34	15	0,018	0,013	0,006	1,271	2,009	0,006	79	5	0,006	0,005	0,001	0,225	0,050	0,001
35	8	0,010	0,012	-0,003	0,515	0,448	0,003	80	3	0,004	0,005	-0,002	0,459	0,482	0,002
36	13	0,016	0,012	0,004	0,848	1,005	0,004	81	7	0,008	0,005	0,003	0,997	1,518	0,003
37	10	0,012	0,012	0,000	0,133	0,018	0,000	82	5	0,006	0,005	0,001	0,068	0,094	0,001
38	9	0,011	0,011	0,000	0,112	0,012	0,000	83	2	0,002	0,005	-0,003	0,873	1,235	0,003
39	10	0,012	0,011	0,001	0,132	0,088	0,001	84	4	0,005	0,005	0,000	0,124	0,015	0,000
40	12	0,014	0,011	0,004	0,884	1,097	0,004	85	5	0,006	0,005	0,001	0,144	0,150	0,001
41	10	0,012	0,010	0,002	0,285	0,206	0,002	86	5	0,006	0,005	0,001	0,169	0,171	0,001
42	2	0,002	0,010	-0,008	2,060	4,934	0,008	87	3	0,004	0,005	-0,001	0,302	0,300	0,001
43	7	0,008	0,010	-0,002	0,268	0,194	0,002	88	4	0,005	0,005	0,000	0,031	0,001	0,000
44	8	0,010	0,010	0,000	0,029	0,001	0,000	89	2	0,002	0,005	-0,002	0,759	1,013	0,002
45	7	0,008	0,010	-0,001	0,144	0,103	0,001	90	7	0,008	0,005	0,004	1,271	2,305	0,004
46	4	0,005	0,009	-0,005	1,168	1,802	0,005	91	2	0,002	0,005	-0,002	0,723	0,948	0,002
47	4	0,005	0,009	-0,004	1,121	1,684	0,004	92	1	0,001	0,005	-0,003	1,214	2,145	0,003
48	6	0,007	0,009	-0,002	0,337	0,270	0,002	93	4	0,005	0,005	0,000	0,079	0,006	0,000
49	6	0,007	0,009	-0,002	0,285	0,220	0,002	94	2	0,002	0,005	-0,002	0,671	0,856	0,002
50	9	0,011	0,009	0,002	0,519	0,496	0,002	95	4	0,005	0,005	0,000	0,121	0,015	0,000
51	8	0,010	0,008	0,001	0,197	0,148	0,001	96	3	0,004	0,005	-0,001	0,118	0,142	0,001
52	12	0,014	0,008	0,006	1,784	3,873	0,006	97	1	0,001	0,004	-0,003	1,142	1,959	0,003
53	6	0,007	0,008	-0,001	0,086	0,077	0,001	98	1	0,001	0,004	-0,003	1,128	1,925	0,003
54	4	0,005	0,008	-0,003	0,820	1,023	0,003	99	5	0,006	0,004	0,002	0,467	0,531	0,002
N	828							χ^2 Real χ^2 Limit						106,65 112,02	MDA 0,0032

In the previous Table, the 1st and 6th columns refer to the first two digits of the values; the 2nd and 7th columns correspond to the sum of the items presenting the first two digits shown in the 1st and 6th columns; the 3rd and 8thcolumns show the proportions of the calculated Sums in the 2nd and 7th columns, in relation to the summation of all the unitary costs in the spreadsheet; the 4th and 9th columns present the standard frequencies of the NB Law; and the 5th and 10th columns show the difference between the proportions of the Sums and the frequencies of the NB Law.

Table 2
Summation Test for the Maracanã unitary costs

Digit	Sum	Real	Benford	Difference	Digit	Sum	Real	Benford	Difference
10	1.117.783,14	0,002	0,011	-0,009	55	1.393,43	0,000	0,011	-0,011
11	35.228.545,85	0,060	0,011	0,049	56	11.333.846,10	0,019	0,011	0,008
12	9.996,90	0,000	0,011	-0,011	57	9.859,91	0,000	0,011	-0,011
13	1.427.480,57	0,002	0,011	-0,009	58	3.104,83	0,000	0,011	-0,011
14	149.926,09	0,000	0,011	-0,011	59	1.184,83	0,000	0,011	-0,011
15	7.327,10	0,000	0,011	-0,011	60	7.413,72	0,000	0,011	-0,011
16	12.400,03	0,000	0,011	-0,011	61	7.613,46	0,000	0,011	-0,011
17	19.334.196,90	0,033	0,011	0,022	62	626,58	0,000	0,011	-0,011
18	18.810.868,98	0,032	0,011	0,021	63	636.977,00	0,001	0,011	-0,010
19	22.983.744,00	0,039	0,011	0,028	64	2.706,05	0,000	0,011	-0,011
20	20.216.982,58	0,035	0,011	0,024	65	723,09	0,000	0,011	-0,011
21	23.621.379,30	0,040	0,011	0,029	66	8.734,92	0,000	0,011	-0,011
22	22.910.130,11	0,039	0,011	0,028	67	2.028,35	0,000	0,011	-0,011
23	3.365,19	0,000	0,011	-0,011	68	68,92	0,000	0,011	-0,011
24	2.441.496,65	0,004	0,011	-0,007	69	6.930.295,76	0,012	0,011	0,001
25	282.240.352,41	0,483	0,011	0,472	70	2.955,46	0,000	0,011	-0,011
26	3.516,32	0,000	0,011	-0,011	71	1.568,94	0,000	0,011	-0,011
27	3.699,11	0,000	0,011	-0,011	72	2.322,08	0,000	0,011	-0,011
28	629,84	0,000	0,011	-0,011	73	4.482,65	0,000	0,011	-0,011
29	9.100,20	0,000	0,011	-0,011	74	1.632,72	0,000	0,011	-0,011
30	3.041.197,41	0,005	0,011	-0,006	75	3.028,68	0,000	0,011	-0,011
31	1.124,59	0,000	0,011	-0,011	76	3.899,28	0,000	0,011	-0,011
32	32.871.298,35	0,056	0,011	0,045	77	7.708.500,52	0,013	0,011	0,002
33	40.266,66	0,000	0,011	-0,011	78	945,72	0,000	0,011	-0,011
34	3.453.111,19	0,006	0,011	-0,005	79	3.254,91	0,000	0,011	-0,011
35	8.277,56	0,000	0,011	-0,011	80	241,03	0,000	0,011	-0,011
36	14.275,68	0,000	0,011	-0,011	81	3.495,51	0,000	0,011	-0,011
37	5.748,38	0,000	0,011	-0,011	82	1.892,95	0,000	0,011	-0,011
38	40.635,97	0,000	0,011	-0,011	83	919,55	0,000	0,011	-0,011
39	2.169,59	0,000	0,011	-0,011	84	3.373,90	0,000	0,011	-0,011
40	6.314,11	0,000	0,011	-0,011	85	4.275,98	0,000	0,011	-0,011
41	85.478,77	0,000	0,011	-0,011	86	4.330,92	0,000	0,011	-0,011
42	8.514,19	0,000	0,011	-0,011	87	1.054,65	0,000	0,011	-0,011
43	6.138,93	0,000	0,011	-0,011	88	353,98	0,000	0,011	-0,011
44	4.472.961,24	0,008	0,011	-0,003	89	1.789,15	0,000	0,011	-0,011
45	4.557.697,21	0,008	0,011	-0,003	90	4.705,38	0,000	0,011	-0,011
46	605,18	0,000	0,011	-0,011	91	182,34	0,000	0,011	-0,011
47	5.356,92	0,000	0,011	-0,011	92	924,80	0,000	0,011	-0,011
48	48.849.864,06	0,084	0,011	0,073	93	2.895,08	0,000	0,011	-0,011
49	7.018,04	0,000	0,011	-0,011	94	1.034,26	0,000	0,011	-0,011
50	2.718,91	0,000	0,011	-0,011	95	11.507,03	0,000	0,011	-0,011
51	1.805,57	0,000	0,011	-0,011	96	1.154,44	0,000	0,011	-0,011
52	3.010,93	0,000	0,011	-0,011	97	976,28	0,000	0,011	-0,011
53	2.242,23	0,000	0,011	-0,011	98	98,55	0,000	0,011	-0,011
54	584.727.527,67	0,000	0,011	-0,011	99	9.975.775,86	0,017	0,011	0,006
TOTAL					584.727.527,67				

As observed on Table 2, there were peaks as regards the first two digits 11, 17, 18, 19, 20, 21, 22, 25, 32 and 48. It is rather striking to observe the proportion found for the digit 25, which represented 48.3% of the total unitary costs According to the Summation test, the data were not nearly as close to a Benford sequence.

3.3 COMPARISON BETWEEN THE FIRST TWO DIGITS TEST AND THE SUMMATION TEST

Next, the digits detected as critical in the First Two Digits Test and the Summation Test were selected. The two tests results were confronted, with the purpose of confirming the criticality of the digits. This was done by comparing their relative frequency in the spreadsheet with the proportion in material terms. The results are presented on table 3.

Table 3 shows the selected digits from both tests (column 1). Column 2 presents the relative frequencies of such digits in the spreadsheet, based on the "Real" column on Table 1. Column 3 displays the proportions of the digits in the budget spreadsheet, according to the "Real" column on Table 2. Column 4 shows whether or not the criticality of such digits is confirmed.

As observed in the comparison between the Tests, the digits 28 and 42 were excluded from the sample. They were only selected for the First Two Digits Test because of their insufficient frequency in the spreadsheet.

The results of the First Two Digits Test pointed to the digits 11, 16 and 25 (excluding the digits 28 and 42). On the other hand, the Summation Test identified excessive values for the proportions of the digits 11, 17, 18, 19, 20, 21, 22, 25, 32 and 48. It was observed that the digits 11 and 25 were identified in both analyses as excessive and, consequently, were more likely to be overestimated. In the case of an audit, an a priori analysis involving the first two digits 11 and 25 would be recommended, and only then the others should be examined.

3.4 COMPARISON BETWEEN THE RESULTS OF THE NB LAW TEST AND THE TCU ANALYSIS

When comparing the digits 11 and 25 with the overpricing found by TCU, the result shown on the table below was obtained.

Table 3: Comparison between the selected digits in the First Two Digits Test and the Summation Test

Digits	Two First Dig.T.	Summation T.	Critcal Digits
11	0,058	0,060	Yes
16	0,039	0,000	Yes
17	0,031	0,033	Yes
18	0,022	0,032	Yes
19	0,019	0,039	Yes
20	0,022	0,035	Yes
21	0,017	0,040	Yes
22	0,014	0,039	Yes
25	0,029	0,483	Yes
28	0,005	0	No
32	0,016	0,056	Yes
42	0,002	0	No
48	0,007	0,084	Yes

It was observed that seven of the items on the ABC Curve had the digits 11 and 25 as the first two digits of the unitary costs, and that the summation of the overpricing found by TCU for those services totaled R$ 41,601,247.32 It was also noted that the tensioned roof System item had the highest overprice on the ABC Curve and it also represented the most expensive service of the work (R$ 256,714,917.00). Its first digits 2 and 5, in addition to being identified by the Summation Test as materially relevant in the budget, were pointed by the First Two Digits Test as excessively frequent in the spreadsheet.

TCU has found a total overprice of R$ 149,972,318.01. The highest amount - R$ 41,601,247.32 – represented 27.74% of the total overprice.

Following the examination of the two most critical digits, the other digits were also analyzed.

Having Table 5 as basis, 10 more items on the ABC Curve - whose overprice had been pointed out by TCU – were identified. Overpricing was detected for all the first two digits examined, except for 18 and 20. However, services 18.052.222-6 – Information board systems, displays (...) and 18.052.259-6 – Transformers, generating sets, no-breaks and (...), whose unitary costs were, respectively, R$ 20,206,546.09 and R$ 18.600.382.98, were not analyzed by TCU. Therefore, it is not possible to prove whether or not their prices were suitable.

The overprice detected for the items on Table 5 totaled R$ 65,692,812.51, which represented 43.8% of what had been pointed out by the Court of Accounts (R$ 149,972,318.01).

When adding the overpricing for the services on Tables 4 and 5, the total amount was R$ 107,294,059.83, which represented 71.54% of all the overpricing detected by TCU.

4. CONCLUSION

The present study tested the application of the Newcomb-Benford Law, used as a data mining tool, to the unitary costs of the budget spreadsheet of the Maracanã remodeling works. The First Two Digits Test and the Summation Test, both from the Benford Law, were carried out. The Z Tests, the Chi-square Test and the Mean Absolute Deviation were applied in order to measure compliance with the NB Law. The tests, in a general way, showed that the unitary costs marginally complied with the Benford Law.

Table 4: Comparison between the results of the NB Law Tests and the TCU analysis for the digits 11 and 25

Dígitos	Service	Unitary Cost	TCU overprice
25	Tensioned roof system, including metal structure, fully locked cables and "PTFE" membrane, according to Hightex project, ref.jop/juv, for Maracanã Stadium World Cup 2014.	256.714.917,00	26.961.972,80
	Sports furniture. and auditorium furniture for the Maracanã Stadium World Cup 2014 – as specified on proposal by Mackey Indústria de Móveis Ltda	25.518.649,48	4.057.726,58
	Demolition of reinforced concrete structures, using compressed air equipment, except floors or pavements.	258,38	699.201,19
11	Aluminum window frames for the Mário Filho – Maracanã Stadium as specified on Itefal proposal n. 12011-0131-000 and complementary frames.	11.920.282,97	1.935.661,95
	Pre-molded parts for Maracanã grandstand, built in reinforced concrete, including from manufacturing to onsite installation, as specified on budget spreadsheet 001/2011-Maracanã World Cup 2014.	11.771.177,22	3.605.433,06
	Sound system (internal areas and grandstand), for the Mário Filho – Maracanã Stadium, as specified on proposal pp 95981/10rev. by Prosegur Forn. and Inst. (Maracanã Worl Cup 2014).	11.512.097,16	3.748.888,14
	High resistance monolithic floor in Polyurethane flakes 3000 with varnish or satin finish and 4 mm nsbrasil or sim. applied on counter-floor and 3mm. incl. this. Maracanã 2014.f/i.	113,28	592.363,60
Total			41.601.247,32

Table 5: Comparison between the results of the NB Law Tests and the TCU analysis for the remaining digits.

Dígitos	Serviço	Custo unitário	Sobrepreço TCU
16	Wedge wire metal grid lining, mesh 30x100mm h=20cm, primary bar 20x2mm, carbon steel SAE 1006/1020, electrostatic painting finish, supply and installation, for Maracanã World Cup 2014.	163,52	1.514.293,41
17	Limited and public access control system, for Mário Filho Stadium – Maracanã, as specified on proposal pp 95981/10 rev.1 by Prosegur. Forn. and Inst.(World Cup 2014).	17.544.505,41	12.370.588,70
	Soil drilling for the molding of root piles, 410mm diam. for Maracanã work World Cup 2014.	177,96	373.048,34
19	Services (wiring, connectorization and interconnection of softwares, assisted commissioning, startup and operations), as specified on prop. pp 95981/10. Prosegur. For. Inst. Maracanã.	19.081.957,09	4.241.578,74
	Floor covering: polished polar white granite plates, measure (60x60) cm. 2.00cm thick laid with a fine layer of cement over cement, sand and lime mortar, proportion 1:2:2 and white cement grouting.	196,72	435.709,44
	Draining, top soil, natural and artificial grass and irrigation system for the Maracanã Stadium lawn – World Cup 2014, as specified on the Campanelli company proposal.	1.919.536,42	722.897,49
21	Demolition of reinforced concrete roof over Maracanã grandstand (marquee), as specified on budget spreadsheet n002/2009-EMOP.	21.439.722,66	13.464.684,58
22	Recovery of the structure of the Mário Filho Stadium – Maracanã – World Cup 2014, as specified on spreadsheet 004/2011 - EMOP.	22.904.212,57	10.839.664,36
32	Local management for remodeling works with changes and additions for the Mário Filho Stadium, Maracanã World Cup 2014, as specified on spreadsheet n 003/2011 EMOP.	32.863.882,27	12.563.282,35
48	System of central air conditioning and complete mechanic ventilation, for the Mário Filho Stadium – Maracanã World Cup 2014, as specified on proposal PR-078620 (Ambienter) supply and assembly.	48.844.340,89	9.167.065,10
Total		R$ 65.692.812,51	

In the individual analysis of the digits of the unitary costs, the first two digits 11 and 25 were detected on the First Two Digits Test and the Summation Test. Such first two digits were contained in seven of the items analyzed by TCU, including the service with the highest overpricing, the Tensioned Roof System (...). The overprice identified for those items totaled R$ 41,601,247,32 and represented 27.74% of all the overpricing identified by the control agency.

In addition, the Summation Test alone identified over 10 services pointed out by TCU with amounts above the market reference prices. The total amount of overprice for the 17 items detected by the NB Law was R$ 107,294,059.83, which represented 71.54% of the total overprice found by TCU (R$ 149,972,318.01).

This study constitutes only the beginning of a research on the application of the NB Law to public works audit. There was an attempt to present the applicability of such tool to budget spreadsheets, so that future research can further develop effective methods for selecting audit samples, by using the NB Law. Such research could seek to study the possible interdependence between the NB Law Tests proposed in the study in question. Furthermore, other significance levels, such as 0.01 and 0.10 – as opposed to 0.05 – could be adopted for the statistical tests, in order to measure compliance. Another interesting possibility for research would involve testing the variable amount on the budget spreadsheet individually, as an alternative to the variable price, with the purpose of checking the applicability and effectiveness of the NB Law in the identification of possible tendencies, as well as comparing those with the analyses carried out by the control agencies for detecting overprice by quantitative increase.