Abstract

This paper investigates the role of the degree of heterogeneity of central bankers’ preferences in the output-inflation tradeoff. It refines the solution of a game theoretic model of monetary policy when inflation targets are not set by the monetary authority and with uncertainty about the preferences of the central banker. The model’s solution implies that greater dispersion in the distribution of central bankers’ preferences increases the signaling cost of commitment to inflation targets. We empirically investigate the role of political fragmentation as a determinant of the neutral interest rate in Brazil and find that greater fragmentation raises the cost of maintaining price stability.

**JEL Classification:** E52, E58, C72  
**Key words:** Inflation targeting, credibility, central bank heterogeneity, exogenous targets, neutral interest rate, political fragmentation.

1. Introduction

Republican George W. Bush was sworn into office as the 43rd President of the USA on January 20, 2001. He succeeded the highly popular Democrat president Bill Clinton, who ruled the country for 8 consecutive years. During the election year of 2000, the Federal Reserve System, the FED, kept their yearly target federal funds rate at 6.5% from May 16 on. Then, on 2001 the FED initiated a slow reduction in interest rates of 0.5% a month until it reached 4.0% in May 2001\(^1\). The corresponding inflation rates were 3.38% in 2000, 2.83% in 2001 and 1.57% in 2002\(^2\). US GDP growth rates, on the other hand, where 5.5% in 2000, 2.19% in 2001 and 3.76% in 2002\(^3\).

\(^1\) [https://www.federalreserve.gov/monetarypolicy/openmarket.htm](https://www.federalreserve.gov/monetarypolicy/openmarket.htm)  
\(^2\) [http://inflationdata.com/Inflation/Inflation_Rate/HistoricalInflation.aspx](http://inflationdata.com/Inflation/Inflation_Rate/HistoricalInflation.aspx)  
Two years later, Labor Party (PT)’s Lula was sworn into office as the 35th President of Brazil on January 1st, 2003. During the election year of 2002 the Brazilian Central Bank raised interest rates steadily from 18.5% in May to 25% in December. Lula’s government increased further interest rates, keeping it at a very high level of 26.5% until May 2003. The corresponding inflation rates were 12.53% in 2002, 9.3% in 2003 and 7.6% in 2004. Brazilian GDP growth rates, on the other hand, where 3.1% in 2002, 1.2% in 2003 and 5.7% in 2004.

A comparison between the two countries’ situation indicates that in the US the election process had no effect on either the downward trajectory of interest rates or on the downward trajectory of inflation, whereas in Brazil there was a clear surge both in interest rates and inflation around the electoral year. This suggests that the electoral process in Brazil may have had a much higher impact on monetary policy and inflation control in Brazil than in the US.

When we look at the institutional characteristics of both countries at that point in time, we can see, among other differences, that Brazil adopted a full-fledged inflation-targeting regime with a central banker appointed by the president at the beginning of his term, whereas the US still had no explicit targeting regime, with a fixed-term central banker whose term is staggered with the president’s.

One fundamental characteristic of the inflation targeting regime is that inflation targets are announced in advance to society. Therefore, inflation expectations based on the announcements and credibility about the central banker’s ability and willingness to deliver the publicized inflation rate play a crucial role in the workings of the system.

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4 http://www.ibge.gov.br/home/estatistica/indicadores/precos/inpe_ipca/defaultseriesHist.shtm
5 In 2012, the US Federal Reserve started to adopt formal and explicit inflation targets, but the target decision is not exogenous to the FRB.
It has been standard in the theoretic literature to assume that inflation targets are set by the monetary authority. However, analyzing inflation targeting (IT) countries’ monetary institutions, one can easily check that in most cases the central banker does not have the autonomy to set the inflation targets. Indeed, according to Mishkin and Schmidt-Hebbel (2001), only 5 out of 19 IT countries allowed their central bankers to independently choose the inflation targets. In the case of Brazil, for instance, inflation targets are decided and set by the Monetary Policy Council (CMN), comprised of the Finance Minister, the Minister of Budget and Planning and the Central Bank’s governor.

The mere existence of an institutional framework that enforces mutual understanding among potentially conflicting members of the government implies that the standard assumption that central bankers set inflation targets may leave behind important dynamics in monetary policy models.

A clear case of such exogenous determination of the target is Brazil in 2007. In that year inflation was under control at around 3.5% and the official target was set at 4.5%. It was time for the CMN to set the 2009 target, and the president of the Central Bank, Henrique Meirelles, openly advocated for a reduction to 4%. However, the CMN decided to maintain the 4.5% target, which has actually been kept until now.

In order to better understand the monetary equilibrium when the central banker does not set inflation targets, we extend the models of Vickers (1986) and Cukierman and Liviatan (1991) by introducing exogenously determined inflation targets and not requiring that any type of central

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6 In fact, that study appoints Poland as a country in which the Central Bank sets the target independently, which is not formally the case according to Horská, 2001. Therefore, the study appoints 6 rather than 5 countries with Central Bank’s goal autonomy. Moreover, only 7 out of the 25 countries listed in footnote 1 allow their central bankers to independently set the inflation targets.

banker achieve the exact target. These assumptions allow us to analyze the role of inflation targets and credibility in inflation expectations’ formation when society is imperfectly informed about the central banker’s characteristics.

Our main innovation is on the solution of the game. We show that the method Vickers (1986) employs to find sequential equilibria fails to encompass certain central bank choices that cannot be ruled out in a perfect Bayesian equilibrium. We apply Cho and Kreps (1987) intuitive criterion as an equilibrium refinement. Under the intuitive criterion, the model implies that greater heterogeneity in central bankers’ types requires costlier disinflationary policies. It is important to highlight that our results are not a generalization of the ones Vickers obtains.

Equilibrium refinements that eliminate equilibrium multiplicity might be desirable from a theoretical perspective. However, the elimination of all pooling equilibria that results in Spence’s signaling model presented in Cho and Kreps (1987) and in the signaling model of monetary policy presented in Vickers (1986) may not be a socially optimum. From the point of view of society, it is better to produce correct inflation expectations in the first period of a two-period game than in the discounted second period.

The fact that the intuitive criterion fails to eliminate the pooling equilibria in the model presented here implies that the elimination of all pooling equilibria in Spence’s signaling model is not to be indiscriminately evoked for every signaling game. This contrasts with Vickers (1986), who adopts dominance and evokes standard stability results for equilibrium refinement.

The most important implication of the model is that a higher (ex-ante) dispersion\(^8\) in central bankers’ preferences causes a strong-type central banker to be tougher on its delivered inflation rates so as to signal his type to society. Naturally, the fact that a player may overshoot, choosing a

\(^8\) Under a reasonable support of discount factor (i.e., greater than 0.5)
strategy above the efficient threshold is well known since the seminal work of Spence (1973) on education choice; the main contribution of the present paper is to relate the overshooting with the spread of the uncertainty about the central banker’s type and its effect on the cost of signaling. In other words, in countries where different types of central bankers have very distinct preferences for monetary policy, disinflation policies will be costlier.

A straightforward implication of the model is to predict that in countries where policy orientation is more heterogeneous, central bankers with a more dovish yet only privately known policy orientation will need to adopt very tight monetary policies in order to be credible.

We empirically test this result by estimating the influence of ex-ante heterogeneity in policy orientation on the neutral interest rate in Brazil, using Bayesian techniques. To this end, we introduce a variant of the semi-structural model presented in Pescatori and Turunem (2015) and Goldfajn and Bicalho (2011). To the best of our knowledge, we are the first to estimate the neutral interest rate and its determinants for Brazil using Bayesian techniques. We find that, even under loose priors, greater political fragmentation raises neutral rates, which, in other words, implies that it becomes costlier to stabilize inflation.

The paper is organized as follows. Section 2 presents a brief review of the game-theory literature underlying our model. Section 3 builds the game-theoretical model of credibility of an inflation-targeting monetary policy. Section 4 discusses the equilibria. Section 5 presents empirical evidence of the impact of political fragmentation on neutral interest rates. Finally, the last section concludes the paper.

2. A brief review of the literature

The role of inflation expectations in short-run output variations has been widely studied since the seminal works of Kydland and Prescott (1977) and Barro and Gordon (1983a, b). With
the advent of the economics of information, several models have analyzed the effects of asymmetric information on the outcome of the monetary policy game played between the central bank and society.

Canzoneri (1985) presents an infinite repeated game between society and a central bank. At each period $t$, society first sets inflation expectations, and the central banker next chooses inflation. However, realized real inflation in period $t$ is affected by a stochastic component to money demand $\delta_t = \epsilon_t + \varepsilon_t$. The model focuses on imperfect asymmetric information on $\delta$: the central banker observes $\epsilon_t$ before choosing inflation but society only observes $\delta_t$ at the end of the period. Because society does not distinguish between $\epsilon_t$ and $\varepsilon_t$, the central banker can create unexpected inflation and attribute it to the unexpected shock $\varepsilon_t$. The solution to the model follows Green and Porter (1984) and finds a trigger strategy equilibrium in which society sets an inflation threshold so that, if realized inflation is below that threshold society expects the Pareto-superior low inflation, but if realized inflation is above that threshold society expects the higher Nash inflation for a punishment period. The model explains periods of high inflation and low employment (stagflation) triggered by the stochastic component of money demand, rather than by the traditional time inconsistency incentives.

Backus and Drifill (1985) focus on incomplete asymmetric information about the type of the central banker, who could be wet or hard-nosed. A wet central banker cares both about controlling inflation and employment whereas a hard-nosed central banker only cares about controlling inflation. The paper considers a finite horizon game between society—who sets inflation expectations—and the central banker—who chooses inflation—and finds a mixed-strategy partially-pooling equilibrium in which the wet central banker mimics the hard-nosed one with positive
probability. In their model, inflation may be lower than expected in the initial periods of the game and higher in the final period.

Vickers (1986) presents a more general game where all types of central banker care both about low inflation and high employment, but they have different relative preferences for inflation and unemployment. The paper focuses on a signaling, separating equilibrium in which the central banker who most values employment (wet) is not able to mimic the central banker who most values low inflation (dry). The game consists of two periods and in equilibrium there will be a recession in the first period if the central banker is dry and there will be expansion if he is wet. Moreover, there will be no surprises in the last period, as all relevant information becomes public in equilibrium. In that paper, as well as in Backus and Driffill (1985), the central banker cannot commit to an announced target. Therefore, there are no explicit inflation targets.

Cukierman and Liviatan (1991) extend Vickers’s model by letting the central banker announce inflation targets before society sets its inflation expectations, in a two-period setup. In their model, a strong central banker will always achieve the exact announced inflation target, whereas a weak one may deviate from the announced target. Walsh (2001) and Bugarin and Carvalho (2005) analyze the monetary equilibria of an extension of Cukierman and Liviatan’s setup to an infinite game where a central banker has a fixed two-period nonrenewable term of office.

Cukierman and Liviatan (1991), Walsh (2001) and Bugarin & Carvalho (2005) allow for announcements of inflation targets, with the assumptions that the announcement is a strategic variable chosen by the central banker and that the strong central banker always delivers on his announced target. Therefore, there is a somewhat artificial, reduced strategic role for the strong central banker, since she cannot deviate from the announced policy.

In light of that, the novelties of the present paper are threefold. First, it considers exogenous inflation targets in a game-theoretic set-up to explicitly analyze the role of credibility in inflation
targets and the role of heterogeneity in the inflation-output tradeoff. Second, there is no exogenous assumption that one type of central banker must follow a specific target, as it is the case in Cukierman & Liviatan (1991). The third novelty is the use of Cho and Kreps (1987) intuitive equilibrium refinement in monetary policy games.

3. A model of credibility and inflation expectations formation with exogenous inflation targets

We extend the models of Vickers (1986) and Cukierman and Liviatan (1991) by introducing exogenously determined inflation targets and not imposing that any type of central banker achieve the exact target. These assumptions allow us to analyze the role of inflation targets and credibility in inflation expectations’ formation when society is imperfectly informed about the central banker’s characteristics. Our main innovation will be on the solution of the game. In the next section, we argue that Vickers left out possible equilibrium choices with important implications for the model’s predictions and we apply Cho and Kreps (1987)’s intuitive criterion for equilibrium selection.

The generic central banker $i$’s utility function at time $t$ is:  

$$ v(\pi_t, \bar{\pi}_t, \pi_t^e) = -\frac{1}{2}(\pi_t - \bar{\pi}_t)^2 + \lambda (\pi_t - \pi_t^e) $$

(1)

where $\pi_t$ is the inflation rate at time $t$ set by the central banker, $\bar{\pi}_t$ is the inflation target for time $t$ that is exogenously set by the government, and $\pi_t^e$ is market inflation expectation for time $t$.

The parameter $\lambda \geq 0$ reflects the importance the central banker attributes to output expansion above trend levels, which is simplified in this model as the (positive) inflationary surprise, relative to the importance he attributes to reaching the inflation target.

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9 This is the simplest way to introduce the traditional trade-off between inflation and growth and follows the seminal articles by Vickers (1986) and Cukierman and Liviatan (1991). For a more detailed derivation of such a reduced form see, for example, Walsh (2000).
The first term on the right represents the (possibly political) cost the central banker incurs by not achieving the target. In inflation targeting regimes the farther away realized inflation is from the target, the stronger the social reaction to central banker’s policies. In certain countries, this could even lead to appointing a new central banker.\textsuperscript{10} Inflation targeting countries usually adopt target bands that are symmetric around the center of the target. Assuming a cost function that is quadratic in the deviation of inflation from the target is a suitable simplification to the common inflation targeting design.

With only one type of central banker and targets exogenously set, the model will predict an inflation bias. The first order condition yields $\pi_t = \bar{\pi} + \lambda$, which means that the central banker will always inflate above target levels. Assuming that expectations are rational, in this one-period game agents will anticipate the inflationary bias and thus no inflation surprises will arise, as $\pi_t^e = \bar{\pi} + \lambda = \pi_t$.

Let us now allow for two possible types of central bankers, $\mu$ and $\lambda$, $\mu \geq \lambda$, who differ from each other because of the relative importance each one privately attributes to output growth with respect to inflation stabilization. Therefore, a central banker that attributes weight $\lambda$ to output expansion cares relatively more about reaching the exogenous target than the central banker that attributes weight $\mu$, who cares relatively more about generating inflationary surprise. The $\lambda$-type central banker of is said to be \textit{strong}, whereas the $\mu$-type is said to be \textit{weak}.

In a one period game, the outcome will be an inflation rate of $\pi_t^S = \bar{\pi} + \lambda$ for the strong type and $\pi_t^W = \bar{\pi} + \mu$ for the weak type. If society believes that the incumbent is of a strong type

\textsuperscript{10} See New Zealand’s institutional framework in Walsh (1995).
with probability $\rho$, inflation expectations will be a weighted average of inflation rates chosen by
the strong and the weak type: $\pi_i^e = \rho \pi_i^S + (1 - \rho) \pi_i^W = \bar{\pi}_i + \rho \lambda + (1 - \rho) \mu$.

This simple analysis allows us to draw the following preliminary conclusions. If central
bankers cannot pre-commit to an inflation target, and if this target is exogenously set, then inflation
expectations will be biased upwards from the target. Realized inflation will also exceed the target,
even if the central banker is of a strong type. Of course, the weaker the central banker is, the higher
the deviation of realized inflation from targets. However, as expected inflation is an average of
inflation rates optimally chosen by a weak and a strong central banker, realized inflation under a
strong type will be lower than the one expected by society.

Note that inflation targets, in spite of not being fulfilled, have a very important role in this
model. As realized inflation is directly related to them, targets guide inflation expectations, thus
working as a nominal anchor to the economy.

Plugging in realized and expected inflation into strong- and weak-type central bankers’
utilities yields respectively $\nu_i^S = -\frac{1}{2} \lambda^2 - \lambda (1 - \rho) (\mu - \lambda)$ and $\nu_i^W = -\frac{1}{2} \mu^2 + \rho (\mu - \lambda) \mu$. Notice
that both types gain with higher credibility in the central banker, which is modeled here as the
parameter $\rho$, i.e., the higher $\rho$, the more society believes that the central banker is strong. Indeed,
if society attributes a higher probability that the central banker is strong, a strong type benefits from
the reduction in society’s “pessimism”, and the model predicts lower inflation expectations and
weaker recession. Moreover, the weak-type central banker benefits from higher inflationary
surprise.

Let us now allow for a two-period game between society and the central banker. Let the
central banker be chosen at random at the beginning of period 1, according to the distribution $(\rho,$
1−ρ), for a two-period term. A time invariant inflation target is concomitantly set by the Executive branch or the Congress for periods 1 and 2: \( \bar{\pi}_1 = \bar{\pi}_2 = \bar{\pi} \). As before, the central banker may be either weak or strong, and this is his private information. Society will thus form expectations based on its belief on the type of the central banker. After expectations have been formed, the central banker sets the inflation rate for period 1. By observing realized inflation, society updates its belief about the type of the central banker and forms inflation expectations for period 2. After expectations have been formed, the central banker sets inflation for the second period and the game finishes. Society’s payoff is a direct measure of the accuracy of its inflation expectations.

Figure 1 depicts the extensive form of the game. The stochastic determination of the central banker’s type (S: strong, W: weak) is modeled by the use of nature (N) in the top decision node. The dotted straight lines represent information sets for society (Soc). The top dotted straight line indicates that society does not know the central banker’s type when setting inflation expectations in period 1. The one on the bottom indicates that if both central bankers’ types choose the same inflation in period 1 in equilibrium, society cannot identify their types. The curved dotted lines indicate that the central banker (respectively society) has infinitely many possible choices for inflation (respectively, for inflation expectations), only one of which is represented in the game tree.

The next section discusses the model’s equilibria and refinements.
4. Equilibria

4.1. Separating Equilibrium

In the separating perfect Bayesian equilibrium, the weak central banker reveals his type to society at the end of the first period. Therefore, he chooses to inflate at its optimal rate in every
period and inflation surprises occur only in the first period of the game. In this equilibrium, realized inflation in periods 1 and 2 under a weak type central banker is $\pi_1^w = \pi_2^w = \overline{\pi} + \mu$.

On the other hand, a strong central banker may have incentives to deviate from its optimal complete information rate if this is necessary to induce the weak central banker not to mimic his chosen inflation. Let $\pi_1^S$ be the inflation chosen by the strong central banker in period 1. Then, the consistent beliefs society holds in period 2, $\pi_2^e$, are the following: if realized inflation in period 1 is lower than or equal to $\pi_1^S$, then the central banker is strong; if it is above $\pi_1^S$, then the central banker is weak. Moreover, society’s expected inflation in period 1 is $\pi_1^e = \rho \pi_1^S + (1 - \rho)(\overline{\pi} + \mu)$.

We can now characterize the separating equilibria.

**Proposition 1:** In a perfect Bayesian separating equilibrium, if $\frac{\lambda}{\mu} \leq 1 - 2\delta$, then inflation set by the strong type central banker satisfies $\pi_1^S \in \left[ \overline{\pi} + \lambda - (2\delta \lambda (\mu - \lambda))^\frac{1}{2}, \overline{\pi} + \lambda \right]$. Otherwise $\pi_1^S \in \left[ \overline{\pi} + \lambda - (2\delta \lambda (\mu - \lambda))^\frac{1}{2}, \overline{\pi} + \mu - (2\delta \mu (\mu - \lambda))^\frac{1}{2} \right]$.

**Proof:**

In order for the weak central banker not to mimic $S$’s choice, it must be the case that choosing his preferred inflation rate $\pi_1^w = \overline{\pi} + \mu$ and revealing his type to society yields a higher utility than choosing $\pi_1^S$, inducing society to believe he is strong, and gaining from the inflationary surprise at period 2. So the weak central banker will not deviate from the separating equilibrium if and only if

$$v(\pi_1^w, \overline{\pi}, \rho \pi_1^S + (1 - \rho)(\overline{\pi} + \mu)) + \delta v(\pi_2^w, \overline{\pi}, \pi_2^w + \mu) \geq v(\pi_1^S, \overline{\pi}, \rho \pi_1^S + (1 - \rho)(\overline{\pi} + \mu)) + \delta v(\pi_2^S, \overline{\pi}, \pi_2^S + \overline{\lambda})$$

This will be the case if and only if the following condition holds:
\[ \pi^s_1 \leq \bar{\pi} + \mu - \left(2\delta \mu (\mu - \lambda)\right)^{\frac{1}{2}} \]  

(2)

In regard to the strong central banker, any deviation from his optimal complete information policy to signal his type brings forward deeper economic recession. Therefore, in a separating equilibrium he must still be better off choosing \( \pi^s_1 \leq \bar{\pi} + \lambda \). If he chooses \( \pi^s_1 > \bar{\pi} + \lambda \), society infers that the central banker is weak. The strong central banker will thus be better off signaling his type and separating if and only if

\[
\nu(\pi^s_1, \bar{\pi}, \rho \pi^s_1 + (1 - \rho)(\bar{\pi} + \mu)) + \delta \nu(\bar{\pi} + \lambda, \bar{\pi}, \bar{\pi}_2 + \lambda) \geq 
\nu(\pi_1 + \lambda, \bar{\pi}, \rho \pi^s_1 + (1 - \rho)(\bar{\pi} + \mu)) + \delta \nu(\bar{\pi}_2 + \lambda, \bar{\pi}, \bar{\pi}_2 + \mu)
\]

and this implies that the following condition should hold in the separating equilibrium:

\[ \pi^s_1 \geq \bar{\pi} + \lambda - \left(2\delta \mu (\mu - \lambda)\right)^{\frac{1}{2}} \]  

(3)

It is straightforward to check that \( \bar{\pi} + \lambda - \left(2\delta \mu (\mu - \lambda)\right)^{\frac{1}{2}} \leq \bar{\pi} + \mu - \left(2\delta \mu (\mu - \lambda)\right)^{\frac{1}{2}} \).

Therefore, there is a range of values for \( \pi^s_1 \) compatible with a separating perfect Bayesian equilibrium.

Note now that the upper bound on the condition for the weak-type not to deviate from the separating equilibrium is higher than the strong-type optimal complete information choice, i.e.,

\[ \pi + \lambda \leq \bar{\pi} + \mu - \left(2\delta \mu (\mu - \lambda)\right)^{\frac{1}{2}} \]

if and only if \( \frac{\lambda}{\mu} \leq 1 - 2\delta \). Therefore, if this condition is satisfied

\[ \left( \frac{\lambda}{\mu} \leq 1 - 2\delta \right) \]

then inflation choices in the interval \( \pi^s_1 \in \left[ \bar{\pi} + \lambda - \left(2\delta \mu (\mu - \lambda)\right)^{\frac{1}{2}}, \bar{\pi} + \lambda \right] \) are the only strong-type choices to belong to a perfect Bayesian equilibrium.\(^\mathrm{11}\) ■

\(^{11}\) Since for any \( \pi^s_1 \in \left[ \bar{\pi} + \lambda, \bar{\pi} + \mu - \left(2\delta \mu (\mu - \lambda)\right)^{\frac{1}{2}} \right] \) the strong central banker would prefer to choose his optimal complete information inflation \( \bar{\pi} + \lambda \) which would also signal his type.
Vickers claims to adopt a method similar to the one that finds sequential equilibria. Although the structure of our model is a direct generalization of that in Vickers, and Fundenberg and Tirole (1991) show an equivalence of sequential equilibria and perfect Bayesian equilibria for classes of games to which our model belongs, our results are not a generalization of the ones Vickers obtain. We show in the Appendix the possible equilibrium choices that Vickers disregarded in his solution of the game.

We now apply Cho and Kreps (1987) intuitive criterion for equilibrium selection.

**Proposition 2:** If $\frac{\lambda}{\mu} \leq 1 - 2\delta$, the only choice of inflation by the strong central banker that fulfills the intuitive criterion is $\pi^S_1 = \overline{\pi} + \lambda$. Otherwise, $\pi^S_1 = \overline{\pi} + \mu - (2\delta\mu(\mu - \lambda))^{\frac{1}{2}}$.

**Proof:**

If $\frac{\lambda}{\mu} \leq 1 - 2\delta$, the perfect Bayesian equilibria are $\pi^S_1 \in \left[\overline{\pi} + \lambda - (2\delta\lambda(\mu - \lambda))^{\frac{1}{2}}, \overline{\pi} + \lambda - (2\delta\lambda(\mu - \lambda))^{\frac{1}{2}}\right]$. Consider any other choice $\pi^S_1$ in the interval $\pi^S_1 \in \left[\overline{\pi} + \lambda - (2\delta\lambda(\mu - \lambda))^{\frac{1}{2}}, \overline{\pi} + \lambda - (2\delta\lambda(\mu - \lambda))^{\frac{1}{2}}\right]$. If the strong central banker can still convince society that he is strong, he can increase his utility by choosing an inflation rate closer to the right-hand side of the interval. At any point in the interval being analyzed, the weak central banker still prefers not to mimic the strong type’s policy. Therefore, $\pi^S_1 = \overline{\pi} + \lambda$ is the only equilibrium inflation rate not to require costly signaling on the part of the strong central banker, and thus it is the only one to fulfill the intuitive criterion.\(^{12}\)
If \( \frac{\lambda}{\mu} > 1 - 2\delta \), then \( \bar{\pi} + \mu - (2\delta \mu (\mu - \lambda))^{\frac{1}{2}} < \bar{\pi} + \lambda \) and any perfect Bayesian equilibrium will require an inflation rate below the strong type’s preferred policy. In that case, every inflation rate \( \pi_s^i \in \left[ \bar{\pi} + \lambda - (2\delta \lambda (\mu - \lambda))^{\frac{1}{2}}, \bar{\pi} + \mu - (2\delta \mu (\mu - \lambda))^{\frac{1}{2}} \right] \) belongs to a perfect Bayesian equilibrium. However, only the choice \( \pi_s^i = \bar{\pi} + \mu - (2\delta \mu (\mu - \lambda))^{\frac{1}{2}} \) satisfies the intuitive criterion\(^{13} \). ■

Note that \( \bar{\pi} > \bar{\pi} + \mu - (2\delta \mu (\mu - \lambda))^{\frac{1}{2}} = \pi_s^i \) if and only if \( \frac{\lambda}{\mu} < \frac{2\delta - 1}{2\delta} \). Therefore, if \( \frac{\lambda}{\mu} > \frac{2\delta - 1}{2\delta} \), then \( \pi_s^i > \bar{\pi} \), i.e., the inflation level chosen by a strong central banker, although below his preferred level \( (\bar{\pi} + \lambda) \), will still be above the target. On the other hand, if \( \frac{\lambda}{\mu} < \frac{2\delta - 1}{2\delta} \), then \( \pi_s^i < \bar{\pi} \), i.e., in order to signal his type the strong central banker will keep inflation below the target \( \bar{\pi} \). Figure 2 summarizes the present analysis.

\(^{13}\) The argument is the same presented in the previous footnote.
The ratio \( \frac{\lambda}{\mu} \) can be interpreted as the level of homogeneity of a society. Indeed, if \( \lambda \) is very close to \( \mu \), so that the ratio is close to one, there is not much divergence in the way different types of central banker value output relatively to achieving the inflation target. This corresponds to the upper right corner of the figure when the discount factor \( \delta \) is high enough (bigger than 0.5). Conversely, if \( \mu \) is much bigger than \( \lambda \), then different types of central bankers diverge strongly and society is heterogeneous. This last case corresponds to the lower right corner of Figure 2.

This suggests that the greater the heterogeneity of central bankers’ types in a society the more conservative will be the strong central bank’s approach to monetary policy conduct in order to convince society that he really is strong.
4.2. Pooling Equilibrium

In a pooling equilibrium, the weak central banker mimics the strong type in the first period of the game. As society observes a first-period rate of inflation that does not allow it to infer which type of central banker is in office, expectations for the second period will be a weighted average of likely inflation rates: \( \pi^e_2 = \rho \pi^S_2 + (1 - \rho) \pi^W_2 = \bar{\pi} + \rho \lambda + (1 - \rho) \mu \). Let \( \pi^p_1 \) be inflation chosen by both types of central bankers in period 1. Then, society will anticipate that actual inflation rate and set:

\[
\pi^e_1 = \pi^S_1 = \pi^W_1 = \pi^p_1.
\]

The consistent beliefs in period 2 are as follows: if the realized inflation in period 1 is lower than or equal to \( \pi^p_1 \), then there is no updating in beliefs, i.e., society still believes that the central banker is strong with the same probability \( \rho \); if it is above \( \pi^p_1 \), then society concludes the central banker is weak. We now characterize the regions for pooling to occur.

**Proposition 3:** If \( \frac{\lambda}{\mu} < 1 - 2\delta \rho \), there will be no pooling equilibrium. On the other hand, if \( \frac{\lambda}{\mu} \geq 1 - 2\delta \rho \), then any inflation level \( \pi^p_1 \in \left[ \bar{\pi} + \mu - (2\delta \mu \rho(\mu - \lambda))^\frac{1}{2}, \bar{\pi} + \lambda \right] \) corresponds to a perfect Bayesian pooling equilibrium.

**Proof:**

Given these beliefs, there cannot be a pooling equilibrium with \( \pi^p_1 > \bar{\pi} + \lambda \), as the strong central banker would prefer to choose \( \pi^S_1 = \bar{\pi} + \lambda \). Therefore, the equilibrium is \( \pi^p_1 \leq \bar{\pi} + \lambda \).
In a pooling equilibrium, the strong central banker will choose $\pi_1^p$ as long as this gives him a higher utility than selecting his preferred policy $\pi + \lambda$ and allowing society to believe that he is weak. Thus, the strong type will not deviate from the pooling equilibrium if and only if:

\[
\begin{align*}
&v(\pi_1^p, \pi, \rho \pi_1^p + (1 - \rho)(\pi + \mu)) + \delta v(\pi + \lambda, \pi, \rho(\pi_2 + \lambda) + (1 - \rho)(\pi + \mu)) \geq \\
v(\pi_1 + \lambda, \pi, \rho \pi_1^s + (1 - \rho)(\pi + \mu)) + \delta v(\pi_2 + \lambda, \pi, \pi_2 + \mu)
\end{align*}
\]

and this condition implies that:

\[
\pi_1^p \geq \pi + \lambda - \left(2\delta\lambda\rho\left(\mu - \lambda\right)\right)^\frac{1}{2} \tag{4}
\]

Likewise, the weak central banker will choose not to deviate from the pooling equilibrium if his utility of mimicking the strong type in the first period is higher than the utility of delivering inflation at his optimal discretionary rate in the first period, thus revealing his type. So the weak type will not deviate from the pooling equilibrium if and only if:

\[
\begin{align*}
&v(\pi_1^p, \pi, \rho \pi_1^s + (1 - \rho)(\pi + \mu)) + \delta v(\pi + \mu, \pi, \rho(\pi_2 + \lambda) + (1 - \rho)(\pi + \mu)) \geq \\
v(\pi_1 + \mu, \pi, \rho \pi_1^s + (1 - \rho)(\pi + \mu)) + \delta v(\pi_2 + \mu, \pi, \pi_2 + \mu)
\end{align*}
\]

and this implies that the following condition should be fulfilled:

\[
\pi_1^p \geq \pi + \mu - \left(2\delta\mu\rho\left(\mu - \lambda\right)\right)^\frac{1}{2} \tag{5}
\]

It follows that $\pi + \lambda - \left(2\delta\lambda\rho\left(\mu - \lambda\right)\right)^\frac{1}{2} \leq \pi + \mu - \left(2\delta\mu\rho\left(\mu - \lambda\right)\right)^\frac{1}{2}$. Therefore, both conditions (4) and (5) will be satisfied if and only if $\pi_1^p \geq \pi + \mu - \left(2\delta\mu\rho\left(\mu - \lambda\right)\right)^\frac{1}{2}$. Furthermore, one must have $\pi_1^p \leq \pi + \lambda$. But $\pi + \lambda \geq \pi + \mu - \left(2\delta\mu\rho\left(\mu - \lambda\right)\right)^\frac{1}{2}$ if and only if $\frac{\lambda}{\mu} \geq 1 - 2\delta\rho$. 


Thus, if \( \frac{\lambda}{\mu} < 1 - 2\delta \rho \) there will be no pooling equilibrium. On the other hand, if \( \frac{\lambda}{\mu} \geq 1 - 2\delta \rho \), then any inflation level \( \pi_1^p \in \left[ \bar{\pi} + \mu - (2\delta \mu \rho (\mu - \lambda))^\frac{1}{2}, \bar{\pi} + \lambda \right] \) corresponds to a perfect Bayesian pooling equilibrium. ■

Pooling will be more likely to occur in the following situations: 1) if the difference between the weak and the strong types is not significant (\( \mu \) close to \( \lambda \)), which would correspond to a more homogeneous society; 2) the weak type significantly values the future (\( \delta \) very high, close to 1); and 3) credibility is high (society expects the central banker is of type \( \lambda \) with high probability, i.e., \( \rho \) is high).

Figure 3 adds to Figure 2 the bold dotted line \( \frac{\lambda}{\mu} = 1 - 2\rho \delta \) (with \( \rho < 1/4 \)). The region above that dotted line corresponds to the model’s pooling equilibria.

We employ the intuitive criterion to try to refine the perfect Bayesian pooling equilibria obtained. This results in the following proposition:

**Proposition 4**: The perfect Bayesian pooling equilibrium in Proposition 3 satisfies the intuitive criterion.

**Proof**: 

To apply the intuitive criterion, we first analyze the hypothetical situation in which a central banker can convincingly signal his type by choosing a very low inflation rate in the first period. The question to be posed to find the intuitive equilibria is: under which conditions does the weak central banker refrain from deviating from the pooling equilibrium?
Should the weak central banker not deviate from the pooling equilibrium, he attains utility:

\[
v_N^w = v(\pi^p, \pi, \pi^p) + \delta v(\pi^w, \pi, \rho \pi^s + (1 - \rho)\pi^w) \\
= v(\pi^p, \pi, \pi^p) + \delta v(\bar{\pi} + \mu, \bar{\pi}, \bar{\pi} + \rho \lambda + (1 - \rho)\pi^w) \\
= -\frac{1}{2} (\pi^p - \bar{\pi})^2 - \frac{1}{2} \delta \mu^2 + \delta \rho \mu (\mu - \lambda)
\]

An out-of-equilibrium strategy to the weak central banker would be to choose an inflation rate \(\pi^D < \pi^p\) so low as to convincingly signal to be strong and attain utility:

\[
v_D^w = v(\pi^D, \pi, \pi^p) + \delta v(\pi^w, \pi, \pi^s) \\
= v(\pi^D, \pi, \pi^p) + \delta v(\bar{\pi} + \mu, \bar{\pi}, \bar{\pi} + \lambda) \\
= -\frac{1}{2} (\pi^D - \bar{\pi})^2 + \mu (\pi^D - \pi^p) - \frac{1}{2} \delta \mu^2 + \delta \mu (\mu - \lambda)
\]

Thus, the weak type does not deviate from pooling if and only if \(v_D^w < v_N^w\), which implies:

\[
\mu [\delta (1 - \rho) (\mu - \lambda)] < \left( \bar{\pi} - \frac{\pi^D + \pi^p}{2} \right) (\pi^p - \pi^D)
\]

(6)

If the strong type does not deviate from the pooling equilibrium, his utility is:

\[
v_N^s = v(\pi^p, \pi, \pi^p) + \delta v(\pi^s, \pi, \rho \pi^s + (1 - \rho)\pi^w) \\
= -\frac{1}{2} (\pi^p - \bar{\pi})^2 - \frac{1}{2} \delta \lambda^2 - \delta \lambda (1 - \rho) (\mu - \lambda)
\]

If he deviates to \(\pi^D < \pi^p\) and fully convinces society of his type, his utility is

\[
v_D^s = v(\pi^D, \pi, \pi^p) + \delta v(\pi^s, \pi, \pi^s) \\
= -\frac{1}{2} (\pi^D - \bar{\pi})^2 + \lambda (\pi^D - \pi^p) - \frac{1}{2} \delta \lambda^2
\]

Thus, the strong type deviates to convincingly signal his type iff \(v_D^s > v_N^s\), or yet

\[
\lambda [\delta (1 - \rho) (\mu - \lambda)] > \left( \bar{\pi} - \frac{\pi^D + \pi^p}{2} \right) (\pi^p - \pi^D)
\]

(7)
Note that, for:

i. the weak type central banker not to deviate from the perfect Bayesian pooling equilibrium, and

ii. the strong type central banker to deviate

it must be the case that conditions (6) and (7) are mutually satisfied, which is impossible given that $0 < \lambda < \mu$.

Therefore, whenever the strong type has incentives to deviate to signal that he is strong, the weak type will also follow. As a result, society cannot update its out-of-equilibrium beliefs, and thus the perfect Bayesian equilibrium obtained satisfies the intuitive criterion.

Vickers (1986) also compares payoffs of deviations from the pooling equilibrium, but states that “it can be demonstrated for a large set of parameter values – roughly speaking, when the relevant inflation rates are positive – that for all pooling equilibria there exists an $x$ (inflation rate) satisfying”: “(a) A wet (weak in our terminology) prefers his pooling equilibrium payoff to the payoff that he would obtain if he chose $\pi_t = x$ and were believed to be dry; and (b) A dry (strong in our terminology)’s pooling equilibrium payoff is worse for him than the payoff he would get if he chose $\pi_t = x$ and were believed to be dry”\textsuperscript{14}. As detailed in the Appendix, Vickers’ method fails to consider equilibrium regions that could not be ruled out in a sequential equilibrium approach.

Although eliminating the pooling equilibria by alternative refinements might be desirable from a theoretical standpoint, it may not be a desirable result from a social perspective. If society’s utility is quadratic in the inflationary surprise, which is analogous to assuming that people do their

\textsuperscript{14} Italicized comments are ours.
best to produce their economic forecasts, pooling may be more desirable given that the inflationary surprise, if there is one, is deferred into the future.

**Figure 3: Pooling equilibrium region**

5. **Empirical Evidence: Political fractionalization and neutral interest rates in Brazil**

   One of the possible interpretations of the model is that a more heterogeneous distribution of central bankers’ types makes it costlier for a strong central banker to build credibility since he will have to make greater efforts to differentiate himself from the other possible types.

   In practice, central bankers cannot directly control inflation but only influence it through available instruments. In inflation targeting regimes, the interest rate is the standard instrument to pursue inflation targets. Ex-ante, strong central bankers keep interest rates high enough to anchor
inflation expectations at the target. Our model can thus be interpreted to imply that for a given inflation target, neutral interest rates will be higher when central bank heterogeneity is higher.

The possible types of central bankers in a society are not observable ex-ante, so we proxy the distribution of central bankers’ policy orientations with that of political parties in the Legislative. We build a political competition index based on the Herfindahl–Hirschman Index (HHI) of seats obtained by each political party in the Brazilian Legislative using data from the World Bank 2015 Database of Political Parties in Cruz et al (2016). This HHI can also be interpreted as a variant of Sartori (1975)’s measure of fractionalization ($Frac$) of the political system:

$$Frac_t = 1 - HHIt$$

where $HHIt = \sum_{i=1}^{N} p_{i,t}$, and $p_{i,t}$ is the share of political party $i$ in total seats in the Legislative. Hence, a higher HHI implies that seats are distributed in a smaller number of parties, i.e., the political system is less fractionalized.

We empirically test whether political fractionalization affects the neutral interest rate in Brazil. To this end, we introduce a variant of the semi-structural models of Pescatori and Turunem (2015) and Goldfajn and Bicalho (2011) to estimate neutral interest rates in Brazil and their determinants.

As in Pescatori and Turunem (2015) our concept of neutral interest rate is a measure of the real rate that is consistent with output at potential and inflation at the target. Following and Laubach and Williams (2003) and Pescatori and Turunem (2015), we apply Bayesian methods to jointly estimate the (unobservable) interest rate and its possible (observable) determinants.

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15 For independent or very small political parties, which are aggregated in DPI 2015, we assumed that they had equal number of seats. This has little impact on the HHI of political heterogeneity because of the very small share of these parties in total seats.
The model includes an IS-curve that relates the output gap $x_t$ to interest rate gap (equation 1), a Phillips curve that relates inflation $\pi_t$ to the output gap (equation 2); and an equation for the neutral rate $r^n_t$ and its determinants (equation 3):

$$x_t = a_1 x_{t-1} + a_2 x_{t-2} - a_r (r_{t-1} - r^n_{t-1}) + \epsilon_t^s$$  \hspace{1cm} (1) \\
$$\pi_t = b_1 \pi_{t-1} + b_y x_{t-1} + \epsilon_t^p$$ \hspace{1cm} (2) \\
$$r^n_t = c g_{t-1} + z_t$$ \hspace{1cm} (3)

where $g_t$ is potential output growth, which follows the rule

$$g_t = \rho g_{t-1} g_{t-1} + \rho g_{t-2} g_{t-2} + \epsilon_t^g$$ \hspace{1cm} (4)

and $r_t$ is the deviation of the real policy rate to its steady state, and its level is defined as the difference between the nominal (observable) rate and expected future inflation. The neutral interest rate, $r^n$, is affected by potential output growth and an exogenous autoregressive process $z$ that is intended to identify the impact of possible determinants ($X$) of the neutral rate:

$$z_t = d_1 z_{t-1} + D_X X_t + \epsilon_t^z$$ \hspace{1cm} (5)

We use yearly data from 1996 to 2014, which encompasses the period of price stability. In the set of possible determinants $X_t$ of the neutral rate are macroeconomic controls: foreign direct investment ($fdi_t$), current account deficit ($ccgd$p_t), net consolidated public sector debt ($debt_t$) financial controls (World Bank’s Z-score of default probability in the banking system, $zscore_t$), demographics and social controls (Gini index, $gini_t$, life expectancy, $lifeE_t$, and age dependency, $ageD_t$).

In more details, we estimate the following version of equation (5):
\[ z_t = d_1z_{t-1} + d_{c cgdp} ccgdp_{t-1} - d_{d f dt} dt_{t-1} + d_{g ini} gini_{t-1} - d_{z score} zscore_{t-1} \]
\[ -d_{h hi} hhi_{t-1} + d_{l ife life} E_{t-1} + d_{age age} D_{t-1} \]
\[ + d_{debt debt} t_{t-1} + \epsilon_t^2 \]

where the variables are expressed in (log) deviations from their steady-states, except for the current account deficit, which is the percent deviation from the steady-state.

To the best of our knowledge, we are the first to employ Bayesian techniques to estimate the neutral rate and its determinants in Brazil. For the Brazilian case, Perreli and Roache (2014) and Goldfajn and Bicalho (2011) also use semi-structural models to estimate the determinants of the neutral interest rate. These studies differ from ours in two main respects. First, they employ traditional estimation techniques to estimate the models, with a higher frequency sample that spans a time-period that is nested in ours. Second, they focus on possible macroeconomic determinants of the neutral rate. In our study, we introduce proxies for political fractionalization and demographics, in addition to other controls that are standard in the literature of neutral rates.

Table 1 shows the mean and 90\% confidence interval of the estimated posterior distribution. The parameter we are mostly interested in is the coefficient of political competition in the legislative \(d_{h hi}\) and given the signs in equation 5a, we expect the estimated mean to be positive. A positive relation indicates that more political fractionalization leads to higher neutral rates. We let the prior be a normal distribution so that the optimization algorithm is not constrained to finding a positive sign for the parameter estimate.

\[ \text{\footnotesize 16 We estimate the model in Dynare, with 2 Metropolis Hastings chains of 1,000,000 draws.} \]
Table 1
Estimated posterior distribution

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<thead>
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<th>Source: Authors’ calculations</th>
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<td>Even adopting a loose prior, we find that the mean value of the parameter $d_{hh1}$ in the posterior distribution is positive, and the 90% confidence interval lies in the positive support of the distribution. In other words, during the price stabilization period of Brazilian history, a greater</td>
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multitude of political orientation, which we assume reflects in possible central bankers’ policy orientation, causes neutral interest rates to be higher. We understand that an environment where neutral rates have to be higher is unfavorable for strong central bankers, since it becomes costlier to maintain a contractionist stance of monetary policy.

6. Conclusion

This paper applied Cho and Kreps (1987)’s intuitive criterion on an extended version of Vickers (1986)’s signaling model of monetary policy to investigate the role of uncertainty regarding the type of a central banker on optimal monetary policy and formation of inflation expectations, in an environment where inflation targets are exogenously set by a government agency that is not the central bank. In contrast to Vickers (1986), we find a range of possible pooling equilibria that survive the intuitive criterion.

The model shows that “social stability” has important implications for monetary policy. Under reasonable values of the discount factor, in more heterogeneous societies, monetary policy has to be more restrictive so as to build on credibility. On the other hand, in more homogeneous societies, the very presence of an inflationary bias will not be grounds for such a restrictive monetary policy stance.

We test the model’s implications by estimating the determinants of the neutral interest rate in Brazil using Bayesian techniques. We find that greater political fractionalization raises neutral rates, which implies that it becomes costlier to control inflation.
References


**APPENDIX**

There are two differences between our theoretical model and that of Vickers (1986):

1. In our model, we allow for an explicit inflation target \( \bar{\pi} \) in central bank’s utility function; in Vickers the implied target is zero.
2. In the intertemporal utility, we add a time discount factor \( \delta \) that may take any value between \((0,1]\); in Vickers the implied discount factor is 1.

However, the solutions we find are not an extension of those found in Vickers. Vickers claims to adopt a methodology to find separating and pooling equilibria very similar to the one that finds sequential equilibria. We shall argue below that under the methodology he employed, some equilibrium intervals were improperly disregarded.

Hereafter, we shall use the terminology adopted in our paper.

**Separating equilibria in Vickers**

To find the separating equilibria, Vickers adopts the following procedure:

1. Define \( \tilde{K}_i \) as the lowest level of inflation the central banker \( i \) chooses in the first period such that he is indifferent between
   a. choosing \( \pi_1 = \tilde{K}_i \) and being believed to be dry – in which case \( \pi_2^e = \lambda \) – and
b. choosing $\pi_i = c_i$, where $c_i$ is his optimal discretionary inflation choice, and being
believed to be wet – in which case $\pi_i^e = \mu$.

2. He calculates $K_i$ for each central banker: $K_i = \lambda \left(1 - \sqrt{2\lambda \mu} \right)$ and $K_{ii} = \mu \left(1 - \sqrt{2\mu \lambda} \right)$. The calculations are as follows:

To find $K_i$, Vickers compares the 2-period utility that a generic central banker $i$ obtains in 1.a and 1.b:

$v(K_i, \rho K_i + (1 - \rho)c_i) + v(c_i, \lambda) = v(c_i, \rho K_i + (1 - \rho)c_i) + v(c_i, \mu) \quad \text{(A.1)}$

$\Rightarrow \frac{1}{2} K_i^2 + c_i \left[\rho K_i + (1 - \rho)c_i - K_i\right] + \frac{1}{2} c_i^2 + c_i (\lambda - c_i)$

$= \frac{1}{2} c_i^2 + c_i \left[\rho K_i + (1 - \rho)c_i - c_i\right] + \frac{1}{2} c_i^2 + c_i (\mu - c_i)$

$\Rightarrow (K_i - c_i)^2 = 2c_i (\mu - \lambda)$

Assuming that $\mu \geq \lambda > 0$, the possibility that $K_i = c_i$ should be ruled out as an indifferent choice of inflation, as the term on the right hand side of the last equality cannot be zero. He is thus left with two cases:

i. $K_i - c_i > 0$, in which case $K_i = c_i \left[1 + 2 \left(\frac{\mu - \lambda}{c_i}\right)\right]$

ii. $K_i - c_i < 0$, in which case $K_i = c_i \left[1 - \frac{\mu - \lambda}{c_i}\right]$

The solution Vickers finds suggests that the only possible case to analyze is “ii”, i.e., $K_i < c_i$

However, there is no reason to rule out the possibility that $K_i - c_i > 0$ for the strong type;
in particular, it should be noted that this region encompasses the strong type’s optimal discretionary choice, $\pi^S_1 = \lambda$, as a possible choice for a separating equilibrium.

**Pooling equilibrium in Vickers**

To build the pooling equilibrium, Vickers tries to find an interval for inflation choices that would make a generic central banker $i$ indifferent between:

i. choosing $\pi_1 = L_i$, and the public cannot infer his type, that is, $\pi^e_2 = \bar{c} = \rho \lambda + (1 - \rho)\mu$

ii. choosing $\pi_1 = c_i$, and the public believes that he’s weak, that is, $\pi^e_2 = \mu$

He breaks down the interval into $L^+_i$, which is the highest level of inflation that sustains the central banker’s indifference, and $L^-_i$ the lowest level of inflation to also sustain the indifference.

Using the central bank’s utility, we can express i and ii as follows:

$$\frac{1}{2}L_i^2 + c_i(L_i - L_r) + \frac{1}{2}c_i^2 + c_i(\bar{c} - c_i) = \frac{1}{2}c_i^2 + c_i(L_i - c_i) + \frac{1}{2}c_i^2 + c_i(\mu - c_i)$$

$$\Rightarrow (L_i - c_i)^2 = 2\rho c_i(\mu - \lambda)$$

Two cases arise:

$L_i - c_i > 0$, in which case, $L_i = c_i + \sqrt{2\rho c_i(\mu - \lambda)}$, or

$L_i - c_i < 0$, in which case, $L_i = c_i - \sqrt{2\rho c_i(\mu - \lambda)}$

For Vickers, $L^+_i$ will be obtained when $L_i - c_i > 0$, for every central banker, and $L^-_i$ will be obtained when $L_i - c_i < 0$. Pooling equilibria will be in the region

$L = [L^+_s, L^-_s] \cap [L^+_y, L^-_y]$ when $\frac{\lambda}{\mu} \geq \frac{1 - 4\rho^2}{1 + 4\rho^2}$. 

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However, as we argue in our paper, the pooling equilibrium does not hold when $L_s > c_S$, as, in this case, the strong type will prefer his optimal discretionary choice, $c_S$.