Efficiency in a Monotonic Partnership with Investment: An endogenous implementation of Holmström's Principal

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Abstract

This note presents a model of a partnership that requires initial investment. The production technology depends on partners' effort choices and, possibly, on an exogenous stochastic term. When the technology is monotonic, the paper shows that the need for an investor is a sufficient condition to ensure efficiency. Efficient sharing rules arise endogenously as outcomes of optimal contracts between the investor and the partners. This result illustrates how a Principal-agent structure can naturally arise in a partnership, without changing its basic internal structure. Indeed, the investor emerges as an (external) implementation of a budget-breaker Principal that solves the inefficiency problem inherent to partnerships, as suggested in Holmström (1982).

JEL classification: D2, D8.

Key words: Partnership, investment, contract, free-rider, efficiency.

1. Introduction

A partnership is characterized by a set of agents (the *partners*) having access to a joint production technology. The production output depends on agents' actions (also referred to as effort) and possibly on a stochastic term, and is publicly observable; however, agents' actions are costly and cannot be verified. Partners share the output according to a specified rule. This sharing rule defines an incentive structure for the partnership and determines indirectly which actions the partners will undertake. The partnership problem is to define a sharing rule that induces a Pareto optimal choice of actions. If such a rule cannot be found, the partnership is said to be inefficient.

In a seminal paper, Holmström (1982) presents the first formal proof that monotonic, deterministic partnerships are inefficient. The main rationale for this inefficiency result is a free rider problem. Indeed, each partner bears alone the cost of his effort; however, the result of his effort, the joint production, is shared among all partners. Since efforts are non-observable, each partner has an incentive to marginally diminish his effort level in order to reduce its cost, which results in an (overall) inefficient supply of effort. In light of that result, Holmström suggests that efficiency can only be implemented in a vertical organization, where a principal is present.

This result gave birth to an important line of research that analyzes what variations of his basic

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setup could reverse the inefficiency result. When the one-shot productive relationship is considered in a repeated framework, Radner (1986) proves that efficiency can be enforced if each period's payoffs are not discounted. When the productive technology is stochastic, Williams and Radner (1995) finds necessary conditions for optimality; most importantly, that study points out the relationship between efficiency and asymmetry in partners' production inputs. When negative shares are allowed (fines), Legros and Matsushima (1991) and Legros and Matthews (1993) find necessary and sufficient conditions for optimality in the stochastic and deterministic production technology cases, respectively. When partners' inputs are complementary, Vislie (1994) shows that efficiency is enforceable and conversely, when partners' inputs are symmetric, Dutta and Radner (1994) show that optimal effort levels cannot be enforced.

The present note explores more directly Holmström's result. Instead of trying to detect which new frictions in a partnership may allow for efficiency, it asks how a Principal-agent structure may naturally arise in a horizontal organization like a partnership in order to solve the inefficiency problem. In order words, this note is concerned with the following question: How can an agent who takes the role of Holmström's Principal arise endogenously in a partnership?

Using a contract theoretic approach, the note shows that, if the partnership needs capital in order to produce, then efficiency will be implemented as a solution to an optimal contract between the partners and the agent who lends the capital: the investor. Therefore, in that situation, the investor plays the role of an outside budget-breaker Principal, without transforming the partnership into an employer-employee relationship.

The rest of the note is organized as follows. Section 2 models a partnership with capital needs. As a benchmark, section 3 characterizes the efficient effort level when the actions of the partners are publicly observable. Section 4 proves that an optimal effort level can also be implemented when the action of each partner is private information, showing that the partnership problem can be solved if initial investment is needed. Finally, section 5 presents some concluding remarks.

2. The partnership economy with investment

The economy consists of n+1 agents indexed i=1,…,n+1. Agents 1,2,…,n are called *partners*; agent n+1 is called *investor*. There are two periods, T=1,2. At period 1, the investor is endowed with one unit of capital and the partners are endowed with a joint production technology φ, which requires an initial investment of 1 unit of capital in order to be used. The production process takes one time period. Agents derive utility from second period consumption. At time T=1 the investor and the partners decide whether to sign an investment contract. If the contract is signed, 1 unit of capital is invested in the partnership's technology, which produces a monetary output according to φ; at time T=2 the partners pay the investor and share the remaining output. It is assumed, without loss of generality, that the investor has no other investment opportunity; therefore, if a contract is not signed she consumes the equivalent of one unit of capital at T=2.

A probability measure space $(\Omega, \mathcal{F}, P)$ depicts the uncertainty in the model. Here Ω is the set of states of the world, $\mathcal{F}$ is a σ-field of subsets of Ω and the probability P is common knowledge. The production technology is described as follows: each partner i chooses an action a_i in a set $A_i \subseteq \mathbb{R}, i = 1, \dots, n$. Let $A = \prod_{i=1}^{n} A_i$. Thus $a = (a_1, \dots, a_n) \in A$ represents an action profile, i.e., a choice of actions for all partners. Write $a_{-i} = (a_1, \dots, a_{i-1}, a_{i+1}, \dots, a_n)$ and $a =$

(a_i, a_{i-1}), as usual. The production technology φ depends on the partners' choice of actions $a \in A$, and on a stochastic term $\theta \in \Omega$. Here $\varphi: A \times \Omega \to \mathbb{R}_+$ is assumed to be non-negative valued[2], measurable and integrable with respect to P and $\varphi(a,\cdot)$ is assumed to be bounded for every $a \in A$. Furthermore, φ satisfies the following monotonicity condition:

$$(\forall a_i, a_i' \in A_i)(a_i > a_i' \to \varphi(a_i, a_{-i}, \theta) > \varphi(a_i', a_{-i}, \theta)\ P\text{-almost everywhere})$$

Note that in the particular case where φ does not depend on θ, i.e., the deterministic case, this corresponds to the monotonicity condition of Holmström (1982). It simply states that a higher effort level by any partner generates a higher output for almost all states of nature.

At time *T*=2 the state of the world θ is realized and the joint production output $\varphi(a, \theta)$ is observed by all agents. However, each agent's effort level is his private information. We can think of the state of the world θ as a general industry-wide variable that affects all companies, such as shocks in consumer's preferences, consumer's wealth, the appearance of new competing products, all of these affecting the industry-wide demand, or as shocks in input prices that affect industry-wide production costs, for example.

By taking action $a_i \in A_i$, partner *i* incurs a utility cost $\nu_i(a_i)$, $\nu_i: A_i \to \mathbb{R}_+$. Partners' utilities are separable in money and action cost, and are linear in money. Thus, if partner *i* takes action a_i and receives shares y_i of the joint production y, her (*ex post*) utility is $u_i(y_i, a_i) = y_i - \nu_i(a_i), i = 1, \dots, n$. Finally, the investor's (*ex post*) utility is $y_{n+1} - 1$ where y_{n+1} is the investor's share of the partnership production y, if a contract is signed at period *T*=1. Her utility is 1 if no contract is agreed upon.

Naturally, we assume that there exists at least one effort profile $a \in A$ such that $E_\theta \varphi(a, \theta) - \sum_{i=1}^{n} \nu_i(a_i) \geq 1$, where E_θ is the expected value operator. That is to say, it is socially efficient for the partnership to produce if it can solve the free rider problem.

The present note characterizes an optimal contract between the investor and the partners in terms of the action profile $a \in A$ that the contract induces. As a benchmark for future comparison, the next section analyzes the perfect monitoring case, in which the actions of the partners are publicly observable.

3. Perfect monitoring

Suppose, first, that the actions of the partners are publicly observable. Then a contract between the *n*+1 agents can be made conditional on each agent's effort choices. Denote such a contract as $\sigma: A \times \varphi(A \times \Omega) \to \mathbb{R}^{n+1}$ where $\sigma_i(a, \phi)$ is the share of agent i if the action profile chosen by the partners is a and the realized state of nature is θ. Given σ, the (interim) von Neumann-Morgenstern expected utility functions of the agents can be written as $U_i(a) = E_\theta \sigma_i\big(a, \varphi(a, \theta)\big) - \nu_i(a_i),\ i = 1, \dots, n$ and $U_{n+1}(a) = E_\theta \sigma_{n+1}\big(a, \varphi(a, \theta)\big) - 1$ where E_θ is the expected value operator. Therefore, an optimal contract σ solves Problem $\mathcal{P}_1$ below, where for each $i = 1, \dots, n,\ \lambda_i \in [0,1]$ is a generic Pareto weight[3] associated to partner *i*.

Problem $\mathcal{P}_1$:

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[2] We use here the notation $\mathbb{R}_+$ for the set of non-negative real numbers.

[3] We assume, naturally, that $\sum_{i=1}^{n} \lambda_i > 0$.

$$\max_{a,\sigma} \sum_{i=1}^{n} \lambda_i \left[E_\theta \sigma_i\big(a, \varphi(a,\theta)\big) - \nu_i(a_i) \right]$$

subject to:

(i) $E_\theta \sigma_{n+1}\big(a, \varphi(a,\theta)\big) = 1$

(ii) $E_\theta \sigma_i\big(a, \varphi(a,\theta)\big) - \nu_i(a_i) \geq 0,\ i = 1, \dots, n$

(iii) $\sigma_i\big(b, \varphi(b,\theta)\big) \geq 0,\ i = 1, \dots, n+1,\ \forall b \in A, \forall \theta \in \Omega$

(iv) $\sum_{i=1}^{n+1} \sigma_i\big(b, \varphi(b,\theta)\big) = \varphi(b,\theta), \forall b \in A, \forall \theta \in \Omega$

(v) $E_\theta \sigma_i\big(a, \varphi(a,\theta)\big) - \nu_i(a_i) \geq E_\theta \sigma_i\big(a_i', a_{-i}, \varphi(a,\theta)\big) - \nu_i(a_i'), \forall a_i' \in A_i, i = 1, \dots, n$

Condition (i) is the Pareto constraint: maximize the welfare of the partners while keeping constant the welfare of the investor. Since the investor has no outside opportunities, we normalize her expected return to 1, without loss of generality[4]. Condition (i) is also an (*ex ante*) Individual Rationality constraint (or Participation Constraint) for the investor to be willing to finance the partnership. Condition (ii) is the (*ex ante*) Individual Rationality constraint for the partners. Condition (iii) is a feasibility constraint; it requires that no agent will pay a fine at any effort profile $b \in A$. This natural requirement constitutes a fundamental difference between this study and both Legros and Matsushima (1991) and Legros and Matthews (1993), where fines can be applied. Although fines are natural in economic theory, Limited Liability constraints are often the case in real world legal systems[5]. Condition (iv) is the Budget Balance constraint; it asserts that the output will be completely shared among the agents regardless of the action profile chosen or the realized state of the world[6]. As a consequence, (iv) ensures that σ will not be changed *ex post*. Notice that condition (iv) also ensures that the sharing rule σ is *durable* in the sense of Holmström and Myerson (1983). Finally, condition (v) is the Incentive Compatibility constraint; it implies that, if σ is not changed *ex post*, then no agent has incentive to unilaterally deviate from the optimal action profile.

Note that the function σ that we wish to choose optimally in Problem $\mathcal{P}_1$ must be defined for all possible realizations of the partnership outcome, which depends both on the state of nature and on the partners' effort profile, i.e., its domain is $\varphi(A \times \Omega)$. This wide domain, together with conditions (iii), (iv) and (v), which depend on all possible effort levels, makes it potentially hard to solve the above problem. However, Lemma 1 shows how Problem $\mathcal{P}_1$ can be simplified.

Lemma 1. *Suppose* $\big(a^*, \sigma^*(a^*, .)\big)$ *maximizes the objective function of Problem* $\mathcal{P}_1$ *subject to* (i),

[4] The problem would remain essentially the same had we allotted higher bargaining power to the investor. For example, condition (i) could be replaced by $E_\theta \sigma_{n+1}(a,\theta) = \mu \geq 1$ as long as there exists an effort profile $a \in A$ such that $E_\theta \varphi(a,\theta) - \sum_{i=1}^{n} \nu_i(a_i) \geq \mu$.

[5] Limited liability tends to lead to a reduction in the range of efficient contracts, as presented in Laffont and Martimort (2002, 3.5). However, as we shall prove, this will not be the case in the present framework. For a discussion on limits to liability from the point of view of the Law, see Cooter and Ulen (2008).

[6] This is the very condition that makes typical Holmström-type partnerships inefficient, and calls for the need of a budget-breaker Principal in order to solve the inefficiency. As we shall see, the investor will play that role in the present economic environment.

(ii), $\sigma_i\big(a, \varphi(a,\theta)\big) \geq 0,\ i = 1, \dots, n+1, \forall\theta \in \Omega$ *and* $\sum_{i=1}^{n+1} \sigma_i\big(a, \varphi(a,\theta)\big) = \varphi(a,\theta), \forall\theta \in \Omega$, *i.e.,* σ^* *is defined only on* $\varphi(\{a^*\} \times \Omega)$, *conditions* (iii) *and* (iv) *are required to hold only at the efficient effort profile, and condition* (v) *is dropped. Then,* σ^* *can be extended to a function* σ^{**} *defined on the entire domain* $\varphi(A \times \Omega)$ *in such a way that* $\big(a^*, \sigma^{**}(.,.)\big)$ *is a solution to Problem* $\mathcal{P}_1$.

Proof: Since partners' efforts can be observed, define the extended function $\sigma^{**}: A \times \varphi(A \times \Omega) \to \mathbb{R}^{n+1}$ as follows.

$$\sigma_i^{**}\left(a^*, \varphi(a^*,\theta)\right) = \sigma_i^*\left(a^*, \varphi(a^*,\theta)\right),\ \forall\theta \in \Omega, i = 1, \dots, n+1,$$

$$\sigma_i^{**}\left(b, \varphi(b,\theta)\right) = 0, \forall b \in A, b \neq a^*, \forall\theta \in \Omega, i = 1, \dots, n,$$

$$\sigma_{n+1}^{**}\left(b, \varphi(b,\theta)\right) = \varphi(b,\theta), \forall b \in A, b \neq a^*, \forall\theta \in \Omega.$$

Then, it is straightforward to check that $\big(a^*, \sigma^{**}(.,.)\big)$ is a solution to Problem $\mathcal{P}_1$.

Lemma 1 shows that, since effort is observable, the search for the contract can essentially focus on the optimal effort level. Therefore, in the case of perfect effort observability, an optimal contract can be completely determined if one can solve the following simpler problem, called $\mathcal{P}_2$.

Problem $\mathcal{P}_2$:

$$\max_{a,\sigma(.)} \sum_{i=1}^{n} \lambda_i\big[E_\theta \sigma_i\big(\varphi(a,\theta)\big) - \nu_i(a_i)\big]$$

subject to:

(i) $E_\theta \sigma_{n+1}\big(\varphi(a,\theta)\big) = 1$

(ii) $E_\theta \sigma_i\big(\varphi(a,\theta)\big) - \nu_i(a_i) \geq 0,\ i = 1, \dots, n$

(iii) $\sigma_i\big(\varphi(a,\theta)\big) \geq 0,\ i = 1, \dots, n+1,\ \forall\theta \in \Omega$

(iv) $\sum_{i=1}^{n+1} \sigma_i\big(\varphi(a,\theta)\big) = \varphi(a,\theta), \forall\theta \in \Omega$

The following propositions completely characterize the efforts levels that can be implemented by optimal contracts.

Proposition 1. *Suppose* $\big(a^*, \sigma^*(\cdot)\big)$ *is a solution to problem* $\mathcal{P}_2$. *Then* a^* *maximizes the Pareto function* $f: A \to \mathbb{R}$ *given by* $f(a) = E_\theta\varphi(a,\theta) - \sum_{i=1}^{n} \nu_i(a_i)$ *with* $f(a^*) \geq 1$.

Proof: Step 1. Let us first prove that $f(a^*) \geq 1$.

From (iv), $E_\theta\varphi(a^*,\theta) = E_\theta\sigma_{n+1}^*\big(\varphi(a^*,\theta)\big) + \sum_{i=1}^{n} E_\theta\sigma_i^*\big(\varphi(a^*,\theta)\big)$.

Now, from (i), $E_\theta\varphi(a^*,\theta) = 1 + \sum_{i=1}^{n} E_\theta\sigma_i^*\big(\varphi(a^*,\theta)\big)$.

Subtracting $\sum_{i=1}^{n} \nu_i(a_i^*)$ in both sides of the previous equation yields:

$E_\theta\varphi(a^*,\theta) - \sum_{i=1}^{n} \nu_i(a_i^*) = 1 + \sum_{i=1}^{n}\big[E_\theta\sigma_i^*\big(\varphi(a^*,\theta)\big) - \nu_i(a_i^*)\big]$.

Hence, from (ii), $f(a^*) = E_\theta\varphi(a^*,\theta) - \sum_{i=1}^{n} \nu_i(a_i^*) \geq 1$.

Step 2. Let us prove next that a^* maximizes the Pareto function f.

Suppose, by way of contradiction, that there exists a $b \in A$ satisfying the conditions (i) through (iv) of problem $\mathcal{P}_2$, such that $f(b) > f(a^*)$.

Define the function $\mu: \varphi(\{b\} \times \Omega) \to \mathbb{R}^{n+1}$ as follows.

For $i = 1, \dots n$,

$$\mu_i(\varphi(b,\theta)) = \left[\nu_i(b_i) + \left[E_\theta \sigma_i^*(\varphi(a^*,\theta)) - \nu_i(a_i^*)\right]\frac{f(b)}{f(a^*)} + \frac{1}{n}\left[\frac{f(b)}{f(a^*)} - 1\right]\right]\left(E_\theta \varphi(b,\theta)\right)^{-1}\varphi(b,\theta).$$

and, $\mu_{n+1}(\varphi(b,\theta)) = \left(E_\theta \varphi(b,\theta)\right)^{-1}\varphi(b,\theta)$.

Then, the pair $(b,\mu(.))$ satisfies all the conditions of problem $\mathcal{P}_1$ with $E_\theta \mu_i(\varphi(b,\theta)) - \nu_i(b_i) > E_\theta \sigma_i^*(\varphi(a^*,\theta)) - \nu_i(a_i^*)$ for all $i = 1, \dots, n$.

Therefore, $\sum_{i=1}^n \lambda_i\left[E_\theta \mu_i(\varphi(b,\theta)) - \nu_i(b_i)\right] > \sum_{i=1}^n \lambda_i\left[E_\theta \sigma_i^*(\varphi(a^*,\theta)) - \nu_i(a_i^*)\right]$.

But that contradicts the optimality of $(a^*, \sigma^*(\cdot))$, which concludes the proof.

Proposition 2. *Suppose a^* solves the problem:*

$$\max_a f(a) = E_\theta \varphi(a,\theta) - \sum_{i=1}^n \nu_i(a_i)$$
$$\text{subject to } f(a) \geq 1$$

Then there exists $\sigma^*(\cdot)$ such that $(a^*, \sigma^*(\cdot))$ is a solution to problem $\mathcal{P}_2$.

Proof: Let $\lambda = \max\{\lambda_1, \dots, \lambda_n\}$ and let $j \in \{1, \dots, n\}$ be such that $\lambda_j = \lambda$.

Step 1. Define $\sigma^*: \varphi(\{a^*\} \times \Omega) \to \mathbb{R}^{n+1}$ as follows.

For $i = 1, \dots n, i \neq j$,

$\sigma_i^*(\varphi(a^*,\theta)) = \nu_i(a_i^*)\left(E_\theta \varphi(a^*,\theta)\right)^{-1}\varphi(a^*,\theta)$,

$\sigma_j^*(\varphi(a^*,\theta)) = \left[\nu_j(a_j^*) + f(a^*) - 1\right]\left(E_\theta \varphi(a^*,\theta)\right)^{-1}\varphi(a^*,\theta)$,

and, $\sigma_{n+1}^*(\varphi(a^*,\theta)) = \left(E_\theta \varphi(a^*,\theta)\right)^{-1}\varphi(a^*,\theta)$.

Then, the pair $(a^*, \sigma^*(\cdot))$ satisfies all the conditions of problem $\mathcal{P}_2$ with $E_\theta \sigma_i^*(\varphi(a^*,\theta)) = \nu_i(a_i^*)$ for all $i = 1, \dots, n, i \neq j$, and, $E_\theta \sigma_j^*(\varphi(a^*,\theta)) = E_\theta \varphi(a^*,\theta) - \sum_{i\neq j,\ i=1}^n \nu_i(a_i^*) - 1$. Furthermore, we have: $\sum_{i=1}^n \lambda_i\left[E_\theta \sigma_i^*(\varphi(a^*,\theta)) - \nu_i(a_i^*)\right] = \lambda_j\left[E_\theta \sigma_j^*(\varphi(a^*,\theta)) - \nu_j(a_j^*)\right]$

Step 2. Suppose now, by way of contradiction, that there exists a pair $(b,\mu(.))$ satisfying conditions of problem $\mathcal{P}_2$ with $\sum_{i=1}^n \lambda_i\left[E_\theta \mu_i(\varphi(b,\theta)) - \nu_i(b_i)\right] > \sum_{i=1}^n \lambda_i\left[E_\theta \sigma_i^*(\varphi(a^*,\theta)) - \nu_i(a_i^*)\right]$.

Define the function $\kappa: \Omega \to \mathbb{R}^{n+1}$ as follows.

For $i = 1, \dots n,\ i \neq j$

$\kappa_i(\varphi(b,\theta)) = \nu_i(b_i)\left(E_\theta \varphi(b,\theta)\right)^{-1}\varphi(b,\theta)$,

$\kappa_j\big(\varphi(b,\theta)\big) = \big[\nu_j(b_j) + f(b) - 1\big]\big(E_\theta\varphi(b,\theta)\big)^{-1}\varphi(b,\theta)$,

and, $\kappa_{n+1}\big(\varphi(b,\theta)\big) = \big(E_\theta\varphi(b,\theta)\big)^{-1}\varphi(b,\theta)$.

Then, the pair $\big(b,\kappa(\cdot)\big)$ satisfies all the conditions of problem $\mathcal{P}_2$ with $E_\theta\kappa_i\big(\varphi(b,\theta)\big) = \nu_i(b_i)$ for all $i = 1,\dots,n, i \neq j$, and, $E_\theta\kappa_j\big(\varphi(b,\theta)\big) = E_\theta\varphi(b,\theta) - \sum_{i\neq j,\ i=1}^{n}\nu_i(b_i) - 1$.

But then,

$\lambda_j\big[E_\theta\kappa_j\big(\varphi(b,\theta)\big) - \nu_j(b_j)\big] \geq \sum_{i=1}^{n}\lambda_i\big[E_\theta\mu_i\big(\varphi(b,\theta)\big) - \nu_i(b_i)\big] > \lambda_j\big[E_\theta\sigma_j^*\big(\varphi(a^*,\theta)\big) - \nu_j(a_j^*)\big]$.

Thus, $f(b) = E_\theta\varphi(b,\theta) - \sum_{i=1}^{n}\nu_i(b_i) = E_\theta\kappa_j\big(\varphi(b,\theta)\big) - \nu_j(b_j) + 1 > E_\theta\sigma_j^*\big(\varphi(a^*,\theta)\big) - \nu_j(a_j^*) + 1 = E_\theta\varphi(a^*,\theta) - \sum_{i=1}^{n}\nu_i(a_i^*) = f(a^*)$, which contradicts the optimality of a^*.

Therefore, it must be the case that $\big(a^*,\sigma^*(\cdot)\big)$ is a solution to problem $\mathcal{P}_1$.

Propositions 1 and 2 show that, in the case of perfect monitoring of effort, there is a contract between the partners and the investor that will create the right incentives for an efficient choice of efforts. The fact that efficiency is implementable when there is perfect information is a well-know property of Principal-Agent type of models (Laffont & Martimort, 2002). The new insights of this paper are, on the one hand, to identify it in a partnership model if there is a need for investment, and, on the other hand, to show that this is still true in the case of imperfect information when the partnership production technology is monotonic, as we show next.

4. Imperfect monitoring

Suppose now that the actions of the partners are not observable, but the output of the partnership can be verified. Suppose, furthermore, that the true state of the world is observable. Then a contract between the investor and the partners has to be contingent on the realized output, rather than on the action profile. Therefore, an optimal contract $\sigma\colon\varphi(A\times\Omega)\to\mathbb{R}^{n+1}$ solves the following problem, referred to as $\mathcal{P}_3$.

Problem $\mathcal{P}_3$:

$$\max_{a,\sigma(.)}\sum_{i=1}^{n}\lambda_i\big[E_\theta\sigma_i\big(\varphi(a,\theta)\big) - \nu_i(a_i)\big]$$

subject to:

(i) $E_\theta\sigma_{n+1}\big(\varphi(a,\theta)\big) = 1$

(ii) $E_\theta\sigma_i\big(\varphi(a,\theta)\big) - \nu_i(a_i) \geq 0,\ i = 1,\dots,n$

(iii) $\sigma_i\big(\varphi(b,\theta)\big) \geq 0,\ i = 1,\dots,n+1,\ \forall b\in A, \forall\theta\in\Omega$

(iv) $\sum_{i=1}^{n+1}\sigma_i\big(\varphi(b,\theta)\big) = \varphi(b,\theta), \forall b\in A, \forall\theta\in\Omega$

(v) $E_\theta\sigma_i\big(\varphi(a,\theta)\big) - \nu_i(a_i) \geq E_\theta\sigma_i\big(\varphi(a_i',a_{-i},\theta)\big) - \nu_i(a_i'),\ \forall a_i'\in A_i, i = 1,\dots,n$

The conditions (i)-(v) have the same interpretation as in the previous section. In particular, conditions (i) and (ii) are Individual Rationality constraints, (iii) is a Limited Liability constraint, condition (iv) is the Budget Balance constraint and condition (v) is the Incentive Compatibility constraint. This last constraint is also a Nash equilibrium condition, since it states that no agent has incentive to unilaterally deviate from the optimal effort choice.

Note that the function σ that we wish to choose optimally in Problem $\mathcal{P}_3$ must be defined for all possible realizations of the partnership outcome, which depends both on the state of nature and on the partners' effort profile, i.e., its domain is $\varphi(A \times \Omega)$. This wide domain, together with conditions (iii), (iv) and (v), which depend on all possible effort levels, again, makes it potentially hard to solve the above problem. However, a simplification in the same line of the one undertaken in the perfect information case can also be used in the present case, as shown in Lemma 2 below.

Lemma 2. *Suppose* $(a^*, \sigma^*(.))$ *maximizes the objective function of Problem* $\mathcal{P}_3$ *subject to* (i), (ii), $\sigma_i(\varphi(a,\theta)) \geq 0,\ i = 1, \dots, n+1, \forall \theta \in \Omega$ *and* $\sum_{i=1}^{n+1} \sigma_i(a, \varphi(a,\theta)) = \varphi(a,\theta), \forall \theta \in \Omega$, *i.e.,* σ^* *is defined only on* $\varphi(\{a^*\} \times \Omega)$, *conditions* (iii) *and* (iv) *are required to hold only at the efficient effort profile, and condition* (v) *is dropped. Then,* σ^* *can be extended to a function* σ^{**} *defined on the entire domain* $\varphi(A \times \Omega)$ *in such a way that* $(a^*, \sigma^{**}(.))$ *is a solution to Problem* $\mathcal{P}_3$.

Proof. Suppose, indeed, that we can find an effort profile a^* and a function σ^* defined only on the domain $\varphi(\{a^*\} \times \Omega)$, i.e., only for the outcomes that could be produced if the agents chose the effort level a^*, such that the pair $(a^*, \sigma^*(.))$ maximizes the objective function of Problem $\mathcal{P}_3$ subject to conditions (i), (ii), $\sigma_i(\varphi(a,\theta)) \geq 0,\ i = 1, \dots, n+1,\ \forall \theta \in \Omega$, and, $\sum_{i=1}^{n+1} \sigma_i(\varphi(a,\theta)) = \varphi(a,\theta), \forall \theta \in \Omega$. Then, the contract function σ^* an be extended to a contract σ^{**} defined on the entire domain $\varphi(A \times \Omega)$ simply as follows, where θ is the realized state of the world ϕ is the realized output.

if $\phi = \varphi(a^*, \theta)$ then $\sigma_i^{**}(\phi) = \sigma_i^*(\phi), i = 1, \dots, n$ and $\sigma_{n+1}^{**}(\phi) = 1$,

if $\phi \neq \varphi(a^*, \theta)$ then $\sigma_i^{**}(\phi) = 0, i = 1, \dots, n$ and $\sigma_{n+1}^{**}(\phi) = \phi$.

It can be easily checked that the resulting pair $(a^*, \sigma^{**}(.))$ satisfies conditions (iii), (iv) and (v) of Problem $\mathcal{P}_3$ and, thereby, is a solution to that problem.

Lemma 2 shows that, in order to solve Problem $\mathcal{P}_3$, it is enough to solve the simplified problem below, which we denote by $\mathcal{P}_4$.

Problem $\mathcal{P}_4$:

$$\max_{a,\sigma(.)} \sum_{i=1}^{n} \lambda_i \left[E_\theta \sigma_i(\varphi(a,\theta)) - \nu_i(a_i) \right]$$

subject to:

(i) $E_\theta \sigma_{n+1}(\varphi(a,\theta)) = 1$

(ii) $E_\theta \sigma_i(\varphi(a,\theta)) - \nu_i(a_i) \geq 0,\ i = 1, \dots, n$

(iii) $\sigma_i(\varphi(a,\theta)) \geq 0,\ i = 1, \dots, n+1, \forall\theta \in \Omega$

(iv) $\sum_{i=1}^{n+1} \sigma_i(\varphi(a,\theta)) = \varphi(a,\theta), \forall\theta \in \Omega$

Note now that problems $\mathcal{P}_2$ and $\mathcal{P}_4$ are identical. This is a consequence of the fact that the existence of a budget breaker allows us to adopt the following simple strategy to solve the optimal contract problem: first focus on finding the optimal effort level for each partner, then devising the correct rewards to the partners when they choose that optimal effort level, then, finally, punish them severely when an outcome incompatible with the choice of the optimal effort levels is realized.

Corollary. Consider a partnership endowed with a stochastic production technology that requires initial investment in order to produce. Suppose the technology is a monotonic function of partners' costly effort choices. Then, there exists an optimal contract between an external investor that induces every partner to supply an efficient effort level to production.

5. Conclusion

This note shows that the need for capital is a sufficient condition to ensure efficiency in a partnership. The result is closely related to the previous literature on partnerships in the following ways.

If one redefines the partnership to include the investor as a partner, then one obtains a highly asymmetric organization, where the action of one of the partners —the investor— is observable. This asymmetry in agents' inputs drives the efficiency result. Therefore, the model confirms Williams and Radner (1995)'s insight on the role of asymmetry in solving the partnership problem.

Most importantly, one can regard the investor as an outside agent who wants to lend capital if she expects the partners will choose actions resulting into nonnegative net returns. Therefore, the investor wants to induce the partners to choose efficient actions and assumes the new role of a budget-breaking Principal. Under that perspective, the model implements the Principal-agent structure proposed by Holmström (1982), without changing its horizontal organization. Note that the driving force behind the feasibility result here is the monotonicity of the production technology on effort, the very same condition that allowed Holmström to prove his inefficiency result when no Principal is present.

Therefore, the main contribution of this note is to highlight the fact that the need for capital may bring about a positive externality to the partnership as it allows it to solve the free rider problem by introducing an exogenous, budget-breaking Principal.

Considering that partnerships are not as common as hierarchical structures, this paper suggests the question of what other tensions may lead to inefficiency in a partnership. A possible answer may lies in the assumption of perfect observability of the state of nature in the imperfect information case. Indeed, since the investor does not participate directly in the productive process, it is natural to believe that partners are better informed about the stochastic component of the production technology. If information asymmetries take the form of a costly state verification model in which partners observe the state of nature whereas the investor bears a cost in order to get informed, then an efficient effort level may not be enforceable. An alternative way to introduce imperfect observability of the state of nature is to assume that agents do not observe θ but a signal s which is correlated to θ. A contract will them depend on the signal s and it is interesting to study whether the investor has an incentive to participate and what will be the effort levels of the partners induced by an optimal contract.

Another possible answer is related to the inclusion of dynamics in the model. Indeed, as the static equilibrium is repeatedly implemented, one may expect that the partners will accumulate gains that will eventually exceed the investment needs. Consequently, the necessity of an outside investor would vanish and efficiency would be in jeopardy. An empirical question associated to that dynamic perspective is to investigate whether small companies that start-up as partnerships tend to transform themselves into hierarchical organizations as profits accumulates and the need for the outside investor diminishes.

The study of the robustness of the efficiency result to the inclusion of dynamics as well as the effects of imperfect observability of the state of the world is left as a suggestion for further research.

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