Trading Forward in the Brazilian Electricity Market

Paulo Coutinho

Andre Rossi de Oliveira

The paper models the interaction between a contract and a spot market whose features are borrowed from the Brazilian electricity market. The spot market is modeled as a random mechanism that yields spot prices of electricity. The contract market is comprised of suppliers, consumers, and marketers. Suppliers and consumers are price takers, while marketers have market power. It is shown that, when the number of consumers increases in the contract market, it is possible for the price of the energy they buy forward to decrease, even if there is only a monopolist marketer in the market. Moreover, the quantity of energy traded in the contract market approaches the total amount of energy available in the system (net of energy sold to captive consumers) when the number of marketers increases without bound.

Field of Research: Economics
Keywords: Brazilian electricity sector, forward markets, risk aversion, marketers, Cournot equilibrium
JEL Classification Numbers: L13, L94 G10, C61, C72

Paulo Coutinho
Affiliation: Accounting Department, Universidade de Brasília, Brazil.
Address: Universidade de Brasilia, Campus Darcy Ribeiro, FACE, Departamento de Ciências Contábeis e Atuariais - CCA, Brasilia – D.F. 70910-900, Brazil.
E-mail: coutinho@unb.br

Andre Rossi de Oliveira
Affiliation: Finance and Economics Department, Woodbury School of Business, Utah Valley University.
Address: 800 University Parkway, Woodbury Business School, Utah valley University, Orem, UT 84097.
E-mail: aoliveira@uvu.edu

Trading Forward in the Brazilian Electricity Market

1. Introduction

In this paper we investigate the interaction between forward and spot markets in a model based on the Brazilian electricity market. Although it draws its main features from the Brazilian case, our model is of interest in and of itself, since it brings new elements to the analysis of contract markets, in particular the role played by marketers.

The Brazilian electricity sector underwent two major overhauls in the last two decades. The first started in 1995, when Law # 8.987, known as the "Concessions Law", was passed by the Brazilian Congress. By establishing the legal framework to regulate the concession of public services, it ushered in a new era in the electricity sector in which several distribution and a few generation companies were privatized, a regulator and a system operator were created, and a wholesale market was structured. This new framework was designed to promote competition in generation and commercialization, and provide open access to the transmission and distribution grids, while keeping distribution and transmission under (incentive) regulation.

The second overhaul took place in the wake of an energy crisis in 2001-02 that forced the federal government to take drastic measures to curtail consumption, and the advent of a new government, in 2003, that came to power with a program calling for reform of the electricity sector. Its agenda came to fruition with the introduction of the so-called "New Electricity Sector Model", in 2004.

This "new" model changed several aspects of the original design of the Brazilian electricity market, but it kept open a contract market where free (i.e. not captive) consumers and generators could trade electricity forward. This opened the door for marketers, agents who purchase and resell energy and/or help close deals between buyers and sellers, to enter the market. In Brazil, these marketers can be either independent or affiliated with generators and/or distributors. The presence of marketers is not an exclusive feature of the Brazilian electricity market. Several other markets around the world, like the PJM and the Texas markets in the United States, also have marketers.

There is no shortage of papers in the literature that study the interface between spot and contract markets. Some of them are theoretical papers interested in the general features of this interaction. Others are applications to different product markets like that of electricity. We will review this literature in the next section. Our paper differs from the recent literature in two main respects. First, it does not allow suppliers (generators) to set quantities or submit supply schedules in the spot market. They actually don't have any control over the spot price. Second, both suppliers and consumers of electricity behave competitively in the contract market. This is in contrast with the recent literature on futures and forward markets, which acknowledges that many commodities traded on those markets are not produced competitively, but in several electricity markets the number of suppliers and consumers is actually relatively large. In the Brazilian market, for instance, there are currently 579 free consumers and 495 generators. In our model, all the market power in the contract market belongs to the marketers, another novelty introduced by our paper.

The paper is organized as follows. Section 2 provides a brief literature review, while section 3 describes the main features of the Brazilian electricity market. Section 4 develops the basic framework of analysis and presents the main findings. It is divided into two sub-sections, one that investigates the case of a monopolist marketer, and the other considers the case of several marketers playing a Cournot game. Section 5 concludes and the Appendix presents the main proofs.

2. Literature Review

The seminal result in the theoretical literature about the interaction between spot and forward markets is Allaz and Vila [1993]. They develop a general model (which applies to several situations, not only energy markets) to show that forward markets can emerge even in the absence of uncertainty. Their main model is a two-period game where (duopoly) producers first buy or sell forward (binding and observable) contracts and then, in the second period, play a Cournot game in quantities in a spot market. A key assumption of their model is perfect foresight, which entails no arbitrage, that is, the forward price is equal to the price that will obtain in the spot market. They show first that producers have strong incentives to sell forward part of their production, for when one of them succeeds in being the only producer to trade forward, he greatly benefits from doing so. Trading on the forward market, however, is a prisoners' dilemma for the producers, since both end up worse off when they trade forward. They also show that Cournot spot markets with forward markets are efficient in the limit, as the number of trading periods goes to infinity.

Several later papers change the assumptions of the model used by Allaz and Vila [1993] and show that their main conclusion that forward markets are social desirable even in the absence of uncertainty may not hold. Mahenc and Salanié [2004], for instance, show that in a model with price-setting duopolists with differentiated products, forward trading results in producers buying forward their own production, so that equilibrium prices are increased compared to the case without forward trading. Green and Le Coq [2010] try to answer a different question, namely how the length of contracts affects the possibility of collusion in a repeated price-setting game. They conclude that firms can always sustain some collusive price above marginal cost if they sell the right number of contracts, whatever their discount factor. As the duration of contracts increases, however, collusion becomes more difficult to sustain.

There is also a large chunk of the literature that focuses on the electricity sector. The seminal paper in this area is Green and Newbery [1992], the first one to apply the concept of supply function equilibrium developed by Klemperer and Meyer [1989] to electricity markets. In their model, generators submit a supply schedule of prices for generation and receive the market-clearing price, which varies with demand. They show that the Nash equilibrium in supply schedules yields a high markup on marginal cost and substantial deadweight losses, and use their findings to explain the early outcomes observed in the British electricity spot market.

Powell [1993] models the contract market in Britain, where financial contracts known as "contracts for differences" (CfDs) are traded. Demand for electricity comes mostly from distribution companies with mean-variance utility. Generators are price setters in the contract market and quantity setters in the spot market. His main conclusions are the following: When generators are non-cooperative in both markets, the competitive result (marginal cost pricing and contract price equal to expected spot price) may emerge; when generators collude in both markets, spot prices are above marginal costs, future prices are above expected spot prices, and hedging is only partial; when generators collude only in the contract market, hedging may be lower still (when risk aversion is sufficiently low). Other early contributions to the study of the UK electricity market are von der Fehr and Harbord [1993] and Wolfram [1998].

Green [1999] is another important reference in this literature. He models the electricity market in the UK as a two-stage game of a spot market and a hedging contract (CfDs) market, just like Powell [1993]. Generators strategies in the spot market are different, however. They simultaneously submit supply functions[1] and the Pool (market operator) considers bids in ascending order. The contract price is determined by an arbitrage condition, which states that it must equal the expected spot price, given the number of contracts sold. The main conclusions of the paper are: (a) A firm with "Bertrand" conjectures will cover all of its expected output in the contract market and will sell at marginal costs in both markets; (b) A firm with "Cournot" conjectures will sell no contracts in equilibrium (in the linear model case); more generally, a risk-neutral firm will not want to use the contract market unless this will affect its rivals' strategies; (c) Generators may cover most of their output in the contract market and still raise prices above their marginal costs; (d) If buyers are risk averse, the contract price may exceed the expected spot price, increasing the generators' incentive to sell in the contract market.

More recent contributions to the literature are Bushnell [2007] (US market), Ciarreta and Espinosa [2010] (Spanish market), and Adilov [2010].

The literature on the workings of the Brazilian electricity market is mostly in Portuguese and doesn't go much beyond providing accounts of the historical evolution of the electricity sector and describing the current system. Exceptions are Dutra and Menezes [2005], who study the properties and outcomes of the auctions carried out in the regulated part of the Brazilian contract market, and Wolak [2008], who presents a proposal for short-term price determination in the wholesale market.

3. *The Brazilian electricity market*

One of the main features of the "New Electricity Sector Model", introduced in 2004, is the existence of two separate energy trading environments. In the first one, named the Regulated Contracting Environment (RCE)[2], energy is sold by electric utilities, independent power producers, self-generators and power marketers, and the only buyers are distribution companies, who are required to contract their entire forecast demand for captive consumers. Contracts are auctioned off over time with delivery dates of one, three, and five years after the date of the auction, and separate auctions for "new" and "existing" electricity[3]. Contracts for new electricity are longer (duration of more than 15 years) than those for existing electricity (eight years). There are also annual "adjustment" auctions where distribution companies can buy additional energy when their forecasts are off the mark. Marketers are only allowed to participate in these adjustment auctions in the regulated environment.

The second trading environment is called the Free Contracting Environment (FCE), and brings together electric utilities, independent power producers, self-generators, marketers, importers, exporters, and free consumers (those that do not need to buy power from distribution companies, typically industrial and commercial firms). Buyers and sellers are free to enter bilateral contracts and negotiate prices, quantities and delivery dates and conditions. Marketers

[1] Green (1999) works with linear supply functions most of the time.

[2] "Ambiente de Contratação Regulada" and "Ambiente de Contratação Livre", respectively, in Portuguese.

[3] "New" electricity refers to power to be generated by plants yet to be built, and "existing" electricity refers to power generated by existing plants.

can be either independent or affiliated with generators and/or distributors. They may either purchase and resell energy or only help close deals between buyers and sellers.

The FCE, also known as the "free market" in Brazilian electricity sector parlance, has been growing steadily in the past few years. It consisted of around 1,100 free and special[4] consumers in 2011, which accounted for approximately 28% of total consumption in the Brazilian electricity system (ABRACEEL [2011]).

Differences between the energy contracted and the energy effectively produced or consumed by market participants are liquidated in the spot market at the so-called "Liquidation of Differences Price." In contrast with other spot markets around the world, no short-term energy trading takes place in the Brazilian market. It is purely a mechanism for multilateral clearing of energy surpluses or deficits. Generators, in particular, do not decide how much energy to produce. That is determined by the system operator based on a dynamic programming model explained below.

The spot price is computed weekly (by submarket) and is based on the marginal operational cost of the system, with lower and upper bounds set by the regulator. Since the Brazilian system is preponderantly hydroelectric, the spot price is computed by a stochastic dynamic programming algorithm that seeks to find the optimal balance between using water today and storing it for future use. To use as much water as possible today to produce electricity is the best short term solution, but that would increase the likelihood of electricity shortfalls in the future. On the other hand, to conserve water today by keeping reservoirs full is the most reliable solution, but it requires higher thermal generation and, thus, higher electricity costs and prices.

4. Model and findings

In this section, we are interested in investigating the impact of power marketers in an electricity market with the characteristics of the Brazilian market. In order to do that, we need to model two separate but interlinked markets, the contract and the spot market. In addition, we need to take into account the fact that the contract market is actually divided into two sub-markets, a regulated (the RCE) and a free market (the FCE).

Let's start with the contract market. Free consumers do not buy in the regulated market, so we don't need to model any interaction between that and the free market as far as demand is concerned. As for the supply side, we make the assumption that generators sell in both (sub) markets, first in the regulated and then in the free market. When making decisions about how much to sell in the latter, they take their commitments in the regulated market as given. Even though it is not entirely realistic, this assumption makes sense if we take into account the fact that in the Brazilian market generators submit bids in auctions carried out within the RCE, and enter long term contracts with distribution companies if their bids are successful. We intend to investigate the possible opportunities for strategic behavior available to suppliers as a result of their presence in both markets in future work.

The spot market[5] is modeled as a mechanism that yields a random spot price. This price is in turn dependent on the demand for electricity forecast by the system operator. Although not a perfect representation of reality, this way of modeling the spot market bears out the main characteristics of the calculation of the spot price in the Brazilian hydroelectric-dominated

[4] Special consumers are those entitled to buy energy from incentivized sources (wind, small hydroelectric plants, biomass and solar).

[5] Even though technically it is not a market, we will continue to use this term.

system. In addition to the forecast demand, inputs to the algorithm used by the Brazilian system operator to compute the spot price are stochastic variables such as the level of water reservoirs, precipitation, evaporation, and other uses of water (irrigation, water supply etc.)[6].

In what follows, we first study the case where there is a single marketer present in the contract market, and then

4.1 Contract market with one marketer

There are two periods in our model. In period 0, a forward contract market with n electricity suppliers (indexed by *k*), m consumers (indexed by *i*), and one marketer opens. The system operator publicly announces the forecast demand to be used in the calculation of the spot price in period 1[7], and the marketer buys forward contracts from suppliers and sells them to consumers at a premium. The spot market opens in period 1, when differences between observed and contracted quantities of electricity are settled at the spot price. Forward contracts are also settled in period 1.

As mentioned above, the spot market is a random mechanism that yields a spot price p. We model the spot price as a random variable

$$p = a - bQ^e + e, \tag{1}$$

where Q^e is the forecast demand and e is a normally distributed random variable with mean m and variance s^2. Both suppliers and consumers are risk averse and have negative exponential utility functions given by $u(p) = -e^{-ap}$, where p is profit.

The profit function of consumer *i* is given by

$$p_i^c = r_i f_i(R_i) - p(R_i - y_i^c) - q^c y_i^c - c_i f_i(R_i), \tag{2}$$

where q^c is the price of a unit of contracted electricity as quoted by the marketer to the consumer, r_i is the given retail price of its product, R_i is the actual amount of electricity used by the consumer to produce $f_i(R_i)$ units of its product, f_i is its production function, c_i is its constant marginal (and average) production cost, and y_i^c is the quantity it buys forward.

Revenue in (2) is equal to the output the consumer produces from a volume R_i of electricity (recall that a free consumer in the forward market is a producer in its product market) times the retail price of its product. We normalize the marginal production cost to zero, and so the cost part of consumer *i*'s profit includes only the cost of buying energy in the spot market and the cost of buying it in the contract market. Notice that the quantity it buys in the spot market is the difference between how much electricity it actually consumes and how much it buys forward.

Since the consumer's utility function has a negative exponential form, its maximization problem can be expressed in terms of the certainty equivalent measure:

---

[6] For a detailed exposition of the stochastic dual dynamic programming based algorithms used by the Brazilian system operator, see Maceira et al. [2008].

[7] An alternative but equivalent assumption would be that the demand for electricity forecast by market agents (suppliers, consumers, and marketers) coincides with that used by the system operator in the algorithm that determines the spot price.

$$\max E\left(p_i^c\right) - \frac{a_i^c}{2} Var\left(p_i^c\right) \qquad (3)$$

where a_i^s is its coefficient of risk aversion.

Before we can compute the expected value and the variance of consumer *i*'s profit, we need to understand how it forms its expectations about its sales in the product market. It would be impractical to model each consumer's product market, so we assume it can perfectly forecast how much of its product it will be producing and selling in period 1. Hence R_i is given[8], and

$$\begin{aligned} E\left(p_i^c\right) &= r_i f_i\left(R_i\right) - \bar{p}(R_i - y_i^c) - q^c y_i^c \\ Var\left(p_i^c\right) &= Var\left(r_i f_i\left(R_i\right) - p(R_i - y_i^c) - q^c y_i^c\right) = \left(R_i - y_i^c\right)^2 s^2 \end{aligned} \qquad (4)$$

where $\bar{p} = a - bQ^e + m$ is the expected value of the spot price.

The solution to problem (3) can be easily calculated:

$$y_i^c = \frac{\bar{p} - q^c}{a_i^c \sigma^2} + R_i = \left(R_i + \frac{\bar{p}}{a_i^c \sigma^2}\right) - \frac{q^c}{a_i^c \sigma^2} = A_i - B_i q^c, \qquad (5)$$

where $A_i = R_i + \frac{\bar{p}}{a_i^c s^2}$ and $B_i = \frac{1}{a_i^c s^2}$. Notice that $A_i > 0$ and $B_i > 0$.

The supplier is a price taker in both the spot and contract markets. Accordingly, its profit function is given by:

$$p_k^g = p\left(F_k - y_k^g\right) + q^g y_k^g - v_k F_k, \qquad (6)$$

where y_k^g is the quantity of output sold forward, q^g is the unit price of contracted electricity quoted by the marketer to the supplier, F_k is the supplier's actual electricity output net of its sales in the regulated market, and v_k is its constant marginal (and average) cost.

In the formulation above, the supplier needs to know how much it will actually be required to produce by the system operator in period 1 in order to figure out the amount of energy to sell forward. We make the simplifying assumption that suppliers are symmetric and so each will produce a quantity of electricity equal to the forecast demand divided by the number of suppliers. Thus $F_k = \left(Q^e / n\right) - F_k^R$ for all *k*, where F_k^R is the electricity the supplier sold in the regulated market. Moreover, we can, without loss of generality, set marginal costs to zero. Notice that, given the definition of F_k, we have $\sum_{k=1}^{n} F_k = \sum_{k=1}^{n}\left[\left(Q^e/n\right) - F_k^R\right] = Q^e - \sum_{k=1}^{n} F_k^R$, and so $\sum_{k=1}^{n} F_k = \sum_{i=1}^{m} R_i$.

The supplier's problem can now be expressed in terms of the certainty equivalent measure:

$$\max \bar{p}\left(F_k - y_k^g\right) + q^g y_k^g - \frac{a_k^g}{2} Var\left(p\left(F_k - y_k^g\right) + q^g y_k^g\right), \qquad (7)$$

---

[8] An equivalent assumption would be that R_i has mean R_i^e and variance 0.

where a_k^g is the supplier's coefficient of risk aversion.

The solution to this problem is

$$y_k^g = \frac{q^g - \overline{p}}{a_k^g \sigma^2} + F_k = \left(F_k - \frac{\overline{p}}{a_k^g \sigma^2} \right) + \frac{q^g}{a_k^g \sigma^2} = C_k + D_k q^g, \tag{8}$$

where $C_k = F_k - \dfrac{\overline{p}}{a_k^g \sigma^2}$ and $D_k = \dfrac{1}{a_k^g \sigma^2}$. It can be easily seen that $D_k > 0$.

The marketer is a monopolist in the contract market. It quotes a selling price to consumers and a buying price to suppliers. It is risk neutral and thus wants to maximize its profits, given by the spread $d = q^c - q^g$ times the quantity traded y. In our model, all trades go through marketers, and so $y = \sum_{i=1}^{m} y_i^s = \sum_{k=1}^{n} y_k^g$. Since quantity demanded is equal to quantity supplied in the contract market, we have $A - Bq^c = C + Dq^g$, where $A = \sum_{i=1}^{m} A_i = R + \frac{\overline{p}}{\sigma^2} \sum_{i=1}^{m} \frac{1}{a_i^c}$, $B = \sum_{i=1}^{m} B_i = \frac{1}{\sigma^2} \sum_{i=1}^{m} \frac{1}{a_i^c}$, $C = \sum_{k=1}^{n} C_k = F - \frac{\overline{p}}{\sigma^2} \sum_{k=1}^{n} \frac{1}{a_k^g}$, $D = \sum_{k=1}^{n} D_k = \frac{1}{\sigma^2} \sum_{k=1}^{n} \frac{1}{a_k^g}$, $R = \sum_{i=1}^{m} R_i$, and $F = \sum_{k=1}^{n} F_k$.

The marketer solves the following maximization problem:

$$\begin{aligned} &\max \left(q^c - q^g \right) y \\ &s.t. \quad q^c - q^g \geq 0, \end{aligned} \tag{9}$$

The proposition below follows from the solution to (9). The proof can be found in the Appendix.

Proposition 1.1: The equilibrium quantities and prices in a forward market where (a) suppliers and consumers are price takers, (b) the marketer has monopoly power, (c) suppliers have the same coefficient of risk aversion, and (d) consumers have the same coefficient of risk aversion, are the following:

$$\begin{aligned} &q^c = \overline{p} + R \left(\frac{a^c \sigma^2}{2m} \right), \qquad q^g = \overline{p} - F \left(\frac{a^g \sigma^2}{2n} \right), \\ &d = \left(\frac{R \sigma^2}{2} \right) \left(\frac{a^c}{m} + \frac{a^g}{n} \right), \qquad y_i^c = R_i - \frac{R}{2m} \text{ and } y_k^g = F_k - \frac{F}{2n}, \end{aligned} \tag{10}$$

where $a_i^s = a^s \ \forall i = 1, \ldots, m$ *and* $a_k^g = a^g \ \forall k = 1, \ldots, n$.

Upon inspection, we can immediately see that the forward price of energy sold (by suppliers) is lower than the expected spot price, whereas the price of energy bought (by consumers) is higher than the expected spot price. Accordingly, suppliers sell forward less than their (net) production and consumers buy forward less than their consumption of electricity.

Risk-averse agents want to hedge against risk. In our model, they do that in the forward market, and any factor that increases the risk (of being exposed to the spot market) or makes the agent more risk-averse increases its demand for hedging, affecting the forward price accordingly. Therefore the following results should come as no surprise:

(i) The forward price paid by (to) a consumer (supplier) is higher (lower) the more risk averse it is. This makes sense because a more risk-averse agent assigns more value to less exposure to the spot market.

(ii) The forward price paid by (to) a consumer (supplier) is higher (lower) the larger the variance of the spot price, since this means more risk.

(iii) The forward price paid by (to) a consumer (supplier) increases (decreases) with total actual consumption (production) for a fixed number of consumers (suppliers). When average consumption (production) is higher, each consumer (supplier) individually has to trade more energy in the market, and this increases its risk of exposure.

A more interesting result can be obtained by letting m, the number of consumers, increase while total demand for electricity does not change. This corresponds to a situation where some captive consumers migrate from the regulated market to the (free) contract market. Since n doesn't change, and, by definition, $F = R$, the price of energy sold forward decreases after the migration takes place.

On the other hand, the behavior of the price of energy bought forward depends on what happens to the ratio R/m. If it is larger after the migration, then q^c increases. If it is lower, q^c decreases. The latter is a surprising result, since the marketer has monopoly power in the contract market. The explanation is that since average consumption decreases, the average consumer is exposed to less spot price risk. As a consequence, the elasticity of demand for contracts increases, for risk sharing becomes less important to the average consumer.

Let's now turn to the marketer. First notice that spread δ is strictly positive, and, as expected, increases with the degree of risk aversion of suppliers, with that of consumers, and with the variance of the spot price. Moreover, since half of the system's (net) energy is traded in the contract market[9], the marketer's profit is equal to $\pi_d = \delta y = \left(\frac{R\sigma^2}{2}\right)\left(\frac{\alpha^c}{m} + \frac{\alpha^g}{n}\right)\left(\frac{R}{2}\right) = R^2\left(\frac{\sigma^2}{4}\right)\left(\frac{\alpha^c}{m} + \frac{\alpha^g}{n}\right)$. When there is migration to the free market, what happens to its profit depends on the behavior of average consumption.

The next section discusses a model where there is more than one marketer.

---

[9] The first way to see this is: $y \equiv \sum_{k=1}^{n} y_k^g = \sum_{k=1}^{n}\left(F_k - \frac{F}{2n}\right) = F - \frac{F}{2} = \frac{F}{2}$. Another way to see it is:
$y \equiv \sum_{k=1}^{n} y_i^s = \sum_{k=1}^{n}\left(R_i - \frac{R}{2m}\right) = R - \frac{R}{2} = \frac{R}{2} = \frac{F}{2}$.

4.2 Contract market with more than one marketer

According to Proposition 1.1, the monopolist marketer obtains a strictly positive spread and, consequently, makes positive profit through its operations in the forward market. This should entice other firms to enter the market as marketers. The situation where there are many marketers is the focus of this section.

There are now H identical marketers and they play a Cournot game. Marketer *h's* profit function is $\pi_h = (q^c - q^g) y_h^m$, where y_h^m is the quantity of energy traded by marketer *h.* Marketer *h* has to solve the following problem, where $y = \sum_{h=1}^{H} y_h^m$:

$$\begin{aligned} &\max_{y_h^m} \ (q^c - q^g) y_h^m \\ &s.t. \quad y = A - Bq^c = C + Dq^g \end{aligned} \tag{11}$$

This problem is equivalent to

$$\max \left(\frac{A - y}{B} - \frac{y - C}{D} \right) y_h^m \tag{12}$$

where the constraint has already been plugged into the objective function.

Proposition 2.1: The equilibrium quantities and prices in a forward market where (a) generators and suppliers are price takers, (b) there are many marketers who play a Cournot game, (c) all generators have the same coefficient of risk aversion and (d) all suppliers have the same coefficient of risk aversion, are given by:

$$\begin{aligned} &q^c = \bar{p} + \left(\frac{\alpha^c \sigma^2}{m} \right) \left(\frac{R}{H + 1} \right), \quad q^g = \bar{p} - \left(\frac{\alpha^g \sigma^2}{n} \right) \left(\frac{F}{H + 1} \right), \\ &\delta = F \left(\frac{\sigma_r^2}{H + 1} \right) \left(\frac{\alpha_s}{m} + \frac{\alpha_g}{n} \right), \quad y_i^c = R_i - \frac{R}{m(H + 1)}, \\ &y_k^g = F_k - \frac{F}{n(H + 1)}, \quad y_h^m = \frac{F}{H + 1} \ \text{and} \ y = \left(\frac{H}{H + 1} \right) F \end{aligned} \tag{13}$$

As far as how they depend on degrees of risk aversion and variances is concerned, prices and quantities bought and sold forward have similar properties to those they featured in the monopolist market case, so we will not comment on them. As a matter of fact, the results of Proposition 2.1 boil down to those of Proposition 1.1 when $H = 1$. The total amount of electricity traded through forward contracts is again less than the (net) energy available, but no longer exactly equal to half of it.

We turn our attention to the effects of a larger number of marketers on the equilibrium values of the variables. First, it is easy to see that the price of energy bought forward decreases with the number of marketers. That is exactly what a Cournot model should yield: The more marketers there are, the stronger the competition between them, and this drives down the price they charge consumers. In the limit, they can charge no more than the expected spot price.

Analogously, the price of energy sold forward increases with the number of marketers, again as a consequence of the enhanced competition between marketers. In the limit, the expected spot price is achieved. It comes as no surprise that the spread charged by marketers goes to zero as the number of marketers increases without bound.

We can also see immediately upon inspection of the formulas for y_i^c and y_k^g that, as the number of marketers increases, the energy sold forward by a supplier approaches its (net) production, while the energy bought forward by a consumer approaches its actual consumption. This is a trivial consequence of the fact that, since the price paid by consumers decreases and the price received by suppliers increases with the number of marketers, generators and suppliers are faced with stronger incentives to hedge their positions in the contract market.

Finally, we should mention that, as the number of marketers increases, the portfolios held by individual marketers shrink in size and the total amount of energy traded in the contract market moves toward the available (net) energy in the system. This was expected, since both consumers and suppliers are forward trading almost all the energy they need or have, respectively. This indicates that the role played by the spot market tends to diminish due to increasing competition between marketers.

5. *Conclusions*

In this paper, we modeled the interaction between marketers, suppliers, and consumers in an electricity forward market when there are no bids allowed in the spot market. This is essentially how the Brazilian electricity market is set up, with the spot price being one of the outputs of a dynamic programming algorithm whose objective is to find the optimal balance between using water today and storing it for future use.

We first obtain results that are standard in the literature. Forward prices paid by consumers are increasing in their degree of risk aversion and the variance of the spot price, while prices suppliers sell their electricity forward for decrease with those same indicators. We also show that a monopolist marketer will be able to charge prices that yield a positive spread, and that it increases with the risk aversion of suppliers and consumers, as well as with the variance of the spot price.

One of our most important contributions comes from the analysis of what happens when the number of consumers in the contract market increases, which in our model corresponds to a scenario where captive consumers migrate from the regulated market to the contract (free) market and there is no change in the total demand for electricity. We point out that one of the consequences is that the price of energy sold forward decreases, which, other things being equal, hurts suppliers. As for the effect on the price of energy bought forward, it depends on the behavior of average consumption. If average consumption is larger after the migration, then the price consumers pay increases. If average consumption is lower, then it decreases. The second possibility is a non-standard result, since the marketer has monopoly power in the contract market. To understand it, notice that when average consumption decreases the average consumer is exposed to less spot price risk. This means that risk sharing becomes less important to him, and so marketers face increased competition from the spot market, which dilutes their market power. This is an issue that, despite its importance to many electricity markets, is not addressed in the literature. For instance, Green [1999], Allaz and Vila [1993] and Powell [1993] all model the generating sector as a duopoly. As for the demand side of the contract market, Green [1999] assumes buyers determine the market-clearing price, Powell [1993] assumes they set quantities, and Allaz and Vila [1993] models them as speculators.

Another important contribution of our paper is to show that the total amount of energy traded in the contract market approaches the system's available energy (net of regulated trades) as the number of marketers increases. This means that spot markets may become less important when there is increased competition between marketers in the contract market, and is something electricity regulators should certainly reckon with.

Finally, we also show that the price of energy bought forward decreases with the number of marketers. The more marketers there are, the stronger the competition between them, and this drives down the price they charge consumers. In the limit, they can charge no more than the expected spot price. Analogously, the price of energy sold forward increases with the number of marketers, and equals the spot price in the limit.

There are many ways in which our model can be improved. We look forward to the opportunity of investigating issues such as the strategic interaction between free and regulated (where the electricity demanded by captive consumers is traded) contract markets, price competition between marketers, and others.

Appendix

Proof of Proposition 1.1:

Since $A - Bq^c = C + Dq^g$, we can rewrite the problem (9) as

$$\begin{aligned} &\max \left(\frac{A - C}{B} - \frac{D}{B} q^g - q^g \right) \left(C + Dq^g \right) \\ &s.t. \quad q^c - q^g \geq 0 \end{aligned} \tag{14}$$

We will first solve the unconstrained problem and then show that the constraint is satisfied at the optimum.

But first let's show that the objective function is concave. Let

$$T(q^g) \equiv \left(\frac{A - C}{B} - \frac{D}{B} q^g - q^g \right) \left(C + Dq^g \right) = \frac{(A - C)C}{B} + $$

$$+ \frac{(A - C)D}{B} q^g - \frac{DC}{B} q^g - \frac{D^2}{B} (q^g)^2 - Cq^g - D(q^g)^2,$$

Since $D > 0, B > 0$, we have

$$\frac{\partial T}{\partial q^g} = \frac{(A - C)D}{B} - \frac{DC}{B} - \frac{2D^2}{B} q^g - C - 2Dq^g$$

$$\frac{\partial^2 T}{\partial (q^g)^2} = \frac{-2D^2}{B} - 2D < 0.$$

Thus the first order condition is both necessary and sufficient for a maximum. The first order condition for this problem is given by

$$\frac{(A-C)D}{B} - \frac{DC}{B} - \frac{2D^2}{B}q^g - C - 2Dq^g = 0$$
$$\Rightarrow \frac{2D^2}{B}q^g + 2Dq^g = \frac{AD - 2DC - BC}{B}$$

This equation can be solved to obtain

$$q^g = \frac{AD - 2DC - BC}{B\left(\frac{2D^2}{B} + 2D\right)} = \frac{AD - C(2D+B)}{2D(D+B)}, \qquad (15)$$

and so

$$\begin{aligned} q^c &= \frac{A-C}{B} - \frac{D}{B}\left(\frac{AD - C(2D+B)}{2D(D+B)}\right) = \frac{A-C}{B} - \frac{AD - C(2D+B)}{2B(D+B)} \\ &= \frac{2(A-C)(D+B) - AD + C(2D+B)}{2B(D+B)} = \frac{AD + 2AB - BC}{2B(D+B)} \qquad (16) \\ &= \frac{AD + B(2A - C)}{2B(D+B)} \end{aligned}$$

The condition $q^g \le q^c$ is satisfied if

$$\frac{AD - C(2D+B)}{2D(D+B)} \le \frac{AD + B(2A-C)}{2B(D+B)}$$

Since $B > 0$ and $D > 0$, this is equivalent to

$$\begin{aligned} &[AD - C(2D+B)]B \le [AD + B(2A - C)]D \\ \Leftrightarrow\ & ABD - 2BCD - CB^2 < AD^2 + 2ABD - BCD \\ \Leftrightarrow\ & ABD + BCD + AD^2 + CB^2 > 0, \end{aligned}$$

Now let’s use the simplifying assumptions that suppliers have the same coefficient of risk aversion, i.e. $\alpha_k^g = \alpha^g\ \forall k = 1, \ldots, n$, and that consumers also have the same coefficient of risk aversion, i.e. $\alpha_i^c = \alpha^c\ \forall i = 1, \ldots, m$. Then $A = R + \frac{m\bar{p}}{\alpha^c \sigma^2}$, $B = \frac{m}{\alpha^c \sigma^2}$, $C = F - \frac{n\bar{p}}{\alpha^g \sigma^2}$, $D = \frac{n}{\alpha^g \sigma^2}$, and so

$$
\begin{aligned}
ABD + CBD + AD^2 + CB^2 &= \left(R + \frac{m\bar{p}}{a^c s^2}\right)\left(\frac{m}{a^c s^2}\right)\left(\frac{n}{a^g s^2}\right) + \left(F - \frac{n\bar{p}}{a^g s^2}\right)\left(\frac{m}{a^c s^2}\right)\left(\frac{n}{a^g s^2}\right) \\
&\quad + \left(R + \frac{m\bar{p}}{a^c s^2}\right)\left(\frac{n}{a^g s^2}\right)^2 + \left(F - \frac{n\bar{p}}{a^g s^2}\right)\left(\frac{m}{a^c s^2}\right)^2 \\
&= \left(R + F\right)\left(\frac{m}{a^c s^2}\right)\left(\frac{n}{a^g s^2}\right) + \left[\frac{m\bar{p}}{a^c s^2} - \frac{n\bar{p}}{a^g s^2}\right]\left(\frac{m}{a^c s^2}\right)\left(\frac{n}{a^g s^2}\right) + \\
&\quad + \left(R + \frac{m\bar{p}}{a^c s^2}\right)\left(\frac{n}{a^g s^2}\right)^2 + \left(F - \frac{n\bar{p}}{a^g s^2}\right)\left(\frac{m}{a^c s^2}\right)^2 \\
&= \left(R + F\right)\left(\frac{m}{a^c s^2}\right)\left(\frac{n}{a^g s^2}\right) + \left(\frac{m\bar{p}}{a^c s^2}\right)\left(\frac{m}{a^c s^2}\right)\left(\frac{n}{a^g s^2}\right) \\
&\quad - \left(\frac{n\bar{p}}{a^g s^2}\right)\left(\frac{m}{a^c s^2}\right)\left(\frac{n}{a^g s^2}\right) + R\left(\frac{n}{a^g s^2}\right)^2 + \left(\frac{m\bar{p}}{a^c s^2}\right)\left(\frac{n}{a^g s^2}\right)^2 \\
&\quad + F\left(\frac{m}{a^c s^2}\right)^2 - \left(\frac{n\bar{p}}{a^g s^2}\right)\left(\frac{m}{a^c s^2}\right)^2 \\
&= \left(R + F\right)\left(\frac{m}{a^c s^2}\right)\left(\frac{n}{a^g s^2}\right) + R\left(\frac{n}{a^g s^2}\right)^2 + F\left(\frac{m}{a^c s^2}\right)^2 > 0
\end{aligned}
$$

which proves that the restriction is satisfied at the optimum.

Let's go back to condition (15). It can be rewritten as

$$
\begin{aligned}
q^g &= \frac{\left(R + \frac{m\bar{p}}{a^c s^2}\right)\left(\frac{n}{a^g s^2}\right) - \left(F - \frac{n\bar{p}}{a^g s^2}\right)\left(\frac{2n}{a^g s^2} + \frac{m}{a^c s^2}\right)}{2\left(\frac{n}{a^g s^2}\right)\left(\frac{n}{a^g s^2} + \frac{m}{a^c s^2}\right)} \\
&= \frac{\frac{Rn}{a^g s^2} + \frac{mn\bar{p}}{a^c a^g \left(s^2\right)^2} - \frac{2Fn}{a^g s^2} - \frac{Fm}{a^c s^2} + \frac{2n^2\bar{p}}{\left(a^g\right)^2\left(s^2\right)^2} + \frac{mn\bar{p}}{a^c a^g \left(s^2\right)^2}}{2\left(\frac{n}{a^g s^2}\right)\left(\frac{n}{a^g s^2} + \frac{m}{a^c s^2}\right)} \\
&= \frac{\frac{\left(R - 2F\right)n}{a^g s^2} - \frac{Fm}{a^c s^2} + \frac{2mn\bar{p}}{a^c a^g \left(s^2\right)^2} + \frac{2n^2\bar{p}}{\left(a^g\right)^2\left(s^2\right)^2}}{2\left(\frac{n}{a^g s^2}\right)\left(\frac{n}{a^g s^2} + \frac{m}{a^c s^2}\right)} \\
&= \frac{\frac{\left(R - F\right)n}{a^g s^2} - F\left(\frac{n}{a^g s^2} + \frac{m}{a^c s^2}\right)}{2\left(\frac{n}{a^g s^2}\right)\left(\frac{n}{a^g s^2} + \frac{m}{a^c s^2}\right)} + \frac{\frac{2n\bar{p}}{a^g s^2}\left(\frac{m}{a^c s^2} + \frac{n}{a^g s^2}\right)}{2\left(\frac{n}{a^g s^2}\right)\left(\frac{n}{a^g s^2} + \frac{m}{a^c s^2}\right)}
\end{aligned}
$$

$$= \bar{p} + \frac{\left(R - F\right)}{2\left(\frac{n}{a^g s^2} + \frac{m}{a^c s^2}\right)} - F\left(\frac{a^g s^2}{2n}\right) \tag{17}$$

Similarly, (16) can be expressed as

$$\begin{aligned}
q^c &= \frac{\left(R + \frac{m\bar{p}}{a^c s^2}\right)\left(\frac{n}{a^g s^2}\right) + \left(\frac{m}{a^c s^2}\right)\left(2R + \frac{2m\bar{p}}{a^c s^2} - F + \frac{n\bar{p}}{a^g s^2}\right)}{\left(\frac{2m}{a^c s^2}\right)\left(\frac{n}{a^g s^2} + \frac{m}{a^c s^2}\right)} \\
&= \frac{\frac{Rn}{a^g s^2} + \frac{mn\bar{p}}{a^c a^g \left(s^2\right)^2} + \frac{\left(2R - F\right)m}{a^c s^2} + \frac{2m^2\bar{p}}{\left(a^c\right)^2\left(s^2\right)^2} + \frac{mn\bar{p}}{a^c a^g \left(s^2\right)^2}}{\left(\frac{2m}{a^c s^2}\right)\left(\frac{n}{a^g s^2} + \frac{m}{a^c s^2}\right)} \\
&= \frac{R\left(\frac{n}{a^g s^2} + \frac{m}{a^c s^2}\right) + \left(R - F\right)\frac{m}{a^c s^2}}{\left(\frac{2m}{a^c s^2}\right)\left(\frac{n}{a^g s^2} + \frac{m}{a^c s^2}\right)} + \frac{\frac{2m\bar{p}}{a^c s^2}\left(\frac{n}{a^g s^2} + \frac{m}{a^c s^2}\right)}{\left(\frac{2m}{a^c s^2}\right)\left(\frac{n}{a^g s^2} + \frac{m}{a^c s^2}\right)} \\
&= \bar{p} + R\left(\frac{a^c s^2}{2m}\right) + \frac{R - F}{2\left(\frac{n}{a^g s^2} + \frac{m}{a^c s^2}\right)}
\end{aligned} \tag{18}$$

Since $R = F$, we can write $q^c = \bar{p} + R\left(\frac{a^c s^2}{2m}\right)$ and $q^g = \bar{p} - F\left(\frac{a^g s^2}{2n}\right)$. Now plug (18) into (5) to get:

$$\begin{aligned}
y_i^c &= R_i + \frac{\bar{p}}{a^c s^2} - \left(\frac{1}{a^c s^2}\right)\left[\bar{p} + R\left(\frac{a^c s^2}{2m}\right)\right] \\
&= R_i + \frac{\bar{p}}{a^c s^2} - \frac{\bar{p}}{a^c s^2} - \frac{R}{2m} \\
&= R_i - \frac{R}{2m}
\end{aligned}$$

Similarly, plug (17) into (8) to obtain

$$\begin{aligned}
y_k^g &= F_k - \frac{\bar{p}}{a^g s^2} + \left(\frac{1}{a^g s^2}\right)\left[\bar{p} - F\left(\frac{a^g s^2}{2n}\right)\right] \\
&= F_k - \frac{\bar{p}}{a^g s^2} + \frac{\bar{p}}{a^g s^2} - \frac{F}{2n} \\
&= F_k - \frac{F}{2n}
\end{aligned}$$

Finally, the spread can be calculated as

$$\begin{aligned} d = q^c - q^g &= \bar{p} + R\left(\frac{\alpha^c \sigma^2}{2m}\right) - \bar{p} + F\left(\frac{\alpha^g \sigma^2}{2n}\right) \\ &= \left(\frac{\sigma^2}{2}\right)\left(\frac{R\alpha^c}{m} + \frac{F\alpha^g}{n}\right) \end{aligned}$$

which, given that $R = F$, boils down to $d = \left(\frac{R\sigma^2}{2}\right)\left(\frac{\alpha^c}{m} + \frac{\alpha^g}{n}\right) = \left(\frac{F\sigma^2}{2}\right)\left(\frac{\alpha^c}{m} + \frac{\alpha^g}{n}\right)$. $\blacksquare$

Proof of Proposition 2.1:

Problem (12) can be rewritten as

$$\max_{y_h^m} \left(\frac{AD + BC - (D + B)y}{BD}\right) y_h^m \tag{19}$$

The first order condition for this problem is:

$$\begin{aligned} & -\frac{(D + B)}{BD} y_h^m + \frac{AD + BC - (D + B)y}{BD} = 0 \\ \Rightarrow\ & -(D + B)y_h^m + AD + BC - (D + B)y_h^m - (D + B)\sum_{j \neq h} y_j^m = 0 \\ \Rightarrow\ & y_h^m \big(2(D + B)\big) = AD + BC - (D + B)\sum_{j \neq h} y_j^m \\ \Rightarrow\ & y_h^m = \frac{AD + BC}{2(D + B)} - \frac{\sum_{j \neq h} y_j^m}{2} \end{aligned} \tag{20}$$

Since the marketers are symmetric, $y_h^m = \frac{y}{H}$ and $\sum_{j \neq h} y_j^m = \frac{(H - 1)y}{H}$. Therefore

$$\begin{aligned} & y_h^m = \frac{AD + BC}{2(D + B)} - \frac{(H - 1)y}{2H} \\ \Rightarrow\ & y = \frac{H(AD + BC)}{2(D + B)} - \frac{H(H - 1)y}{2H} \\ \Rightarrow\ & y + \frac{(H - 1)}{2} y = \frac{H(AD + BC)}{2(D + B)} \\ \Rightarrow\ & y = \left[\frac{H(AD + BC)}{2(D + B)}\right]\left[\frac{2}{H + 1}\right] = \frac{H(AD + BC)}{(H + 1)(D + B)} \\ & \text{and } y_h^m = \frac{AD + BC}{(H + 1)(D + B)} \end{aligned} \tag{21}$$

Now we plug the formulas for *A*, *B*, *C* and *D* into (21) to get

$$
\begin{aligned}
y_h^m &= \frac{\left(R + \frac{m\bar{p}}{a^c s^2}\right)\frac{n}{a^g s^2} + \frac{m}{a^c s^2}\left(F - \frac{n\bar{p}}{a^g s^2}\right)}{(H+1)\left(\frac{n}{a^g s^2} + \frac{m}{a^c s^2}\right)} \\
&= \frac{\frac{Rn}{a^g s^2} + \frac{mn\bar{p}}{a^c a^g \left(s^2\right)^2} + \frac{Fm}{a^c s^2} - \frac{mn\bar{p}}{a^c a^g \left(s^2\right)^2}}{(H+1)\left(\frac{n}{a^g s^2} + \frac{m}{a^c s^2}\right)} \\
&= \frac{F\left(\frac{m}{a^c s^2} + \frac{n}{a^g s^2}\right)}{(H+1)\left(\frac{n}{a^g s^2} + \frac{m}{a^c s^2}\right)} = \frac{F}{H+1} \text{ and } y = \left(\frac{H}{H+1}\right)F,
\end{aligned}
\qquad (22)
$$

where we used the fact that $R = F$.

The next step is to calculate the prices:

$$
\begin{aligned}
q^c &= \frac{A - y}{B} = \frac{R + \frac{m\bar{p}}{a^c s^2} - \left(\frac{H}{H+1}\right)F}{\frac{m}{a^c s^2}} \\
&= \frac{R a^c s^2}{m} + \bar{p} - \left(\frac{H}{H+1}\right)\left(\frac{a^c s^2}{m}\right)R = \bar{p} + R\left(\frac{a^c s^2}{m}\right)\left(\frac{1}{H+1}\right)
\end{aligned}
\qquad (23)
$$

and

$$
\begin{aligned}
q^g &= \frac{y - C}{D} = \frac{\left(\frac{H}{H+1}\right)F - F + \frac{n\bar{p}}{a^g s^2}}{\frac{n}{a^g s^2}} \\
&= \frac{\frac{n\bar{p}}{a^g s^2} - \left(\frac{1}{H+1}\right)F}{\frac{n}{a^g s^2}} = \bar{p} - F\left(\frac{a^g s^2}{n}\right)\left(\frac{1}{H+1}\right)
\end{aligned}
\qquad (24)
$$

The spread can now be easily calculated:

$$
\begin{aligned}
d &= q^c - q^g = \bar{p} + F\left(\frac{a^c s^2}{m}\right)\left(\frac{1}{H+1}\right) - \bar{p} + F\left(\frac{a^g s^2}{n}\right)\left(\frac{1}{H+1}\right) \\
&= F\left(\frac{s^2}{H+1}\right)\left(\frac{a^c}{m} + \frac{a^g}{n}\right)
\end{aligned}
$$

Quantities contracted by suppliers and consumers can be obtained by using (5), (8), (23) and (24):

$$
\begin{aligned}
y_k^g &= F_k - \frac{\overline{p}}{a^g s^2} + \frac{1}{a^g s^2}\left(\overline{p} - F\left(\frac{a^g s^2}{n}\right)\left(\frac{1}{H+1}\right)\right) \\
&= F_k - \frac{\overline{p}}{a^g s^2} + \frac{\overline{p}}{a^g s^2} - \frac{F}{n(H+1)} = F_k - \frac{F}{n(H+1)}
\end{aligned}
\tag{25}
$$

and

$$
\begin{aligned}
y_i^c &= R_i + \frac{\overline{p}}{a^c s^2} - \frac{1}{a^c s^2}\left(\overline{p} + R\left(\frac{a^c s^2}{m}\right)\left(\frac{1}{H+1}\right)\right) \\
&= R_i + \frac{\overline{p}}{a^c s^2} - \frac{\overline{p}}{a^c s^2} - \frac{R}{m(H+1)} = R_i - \frac{R}{m(H+1)}
\end{aligned}
\tag{26}
$$

Finally, we can check our calculations as follows:

$$
\begin{aligned}
\sum_{k=1}^{n} y_k^g &= \sum_{k=1}^{n}\left(F_k - \frac{F}{n(H+1)}\right) = F - \frac{F}{H+1} = \left(\frac{H}{H+1}\right)F \\
\sum_{i=1}^{m} y_i^c &= \sum_{i=1}^{m}\left(R_i - \frac{R}{m(H+1)}\right) = R - \frac{R}{H+1} = \left(\frac{H}{H+1}\right)R \quad \blacksquare \\
\sum_{h=1}^{H} y_h^m &= \sum_{h=1}^{H} \frac{F}{H+1} = \left(\frac{H}{H+1}\right)F
\end{aligned}
$$

References

ABRACEEL (2011). **Relatório Anual 2011**. Available at http://www.abraceel.com.br/clipping/documentos/detalhes/4620/relatorio-anual-2011.

Adilov, N. (2010). "Bilateral forward contracts and spot prices." *The Energy Journal* **31**(3): 67-81.

Allaz, B., and J. L. Vila (1993). "Cournot competition, forward markets and efficiency." *Journal of Economic Theory* **59**(1): 1-16.

Bushnell, J. (2007). "Oligopoly equilibria in electricity contract markets." *Journal of Regulatory Economics* **32**: 225–245.

Ciarreta, A., and M. P. Espinosa (2010). "Supply function competition in the Spanish wholesale electricity market." *The Energy Journal* **31**(4): 137-157.

Dutra, J., and F. Menezes (2005). "Lessons from the electricity auctions in Brazil." *The Electricity Journal* **18**(10): 11-21.

Green, R. (1996). "Increasing competition in the British electricity spot market." Journal of Industrial Economics **44**: 205-216.

Green, R. (1999). "The electricity contract market in England and Wales." *The Journal of Industrial Economics* **47**(1): 107-124.

Green, R., and C. Le Coq (2010). "The length of contracts and collusion." International Journal of Industrial Organization **28**: 21–29.

Green, R., and D. M. Newbery (1992). "Competition in the British electricity spot market." *Journal of Political Economy* **110**(5): 929-953.

Klemperer, P. D., and M. A. Meyer (1989). "Supply function equilibria in oligopoly under uncertainty." *Econometrica* **57**(6): 1243-1277.

Maceira, M., et al. (2008). "Ten years of application of stochastic dual dynamic programming in official and agent studies in Brazil - Description of the NEWAVE program." Paper presented at the 16th PSCC, Glasgow, Scotland, July 14-18.

Mahenc, P., and F. Salanié (2004). "Softening competition through forward trading." *Journal of Economic Theory* **116**: 282-293.

Powell, A. (1993). "Trading forward in an imperfect market: The case of electricity in Britain." The Economic Journal **103**(March): 444-453.

von der Fehr, N.-H. M., and D. Harbord (1993). “Spot market competition in the UK electricity industry.” *The Economic Journal* **103**: 531-546.

Wolak, F. (2008). “Options for Short-Term Price Determination in the Brazilian Wholesale Electricity Market.” Report Prepared for Câmara de Comercialização de Energia Elétrica (CCEE). Available at http://iis-db.stanford.edu/pubs/22874/brazil_report_ccee_wolak.pdf.

Wolfram, C. D. (1998). “Strategic bidding in a multiunit auction: An empirical Analysis of bids to supply electricity in England and Wales.” RAND Journal of Economics **29**: 703-725.